Mathematica 11.3 Integration Test Results

Test results for the 538 problems in "7.3.4 u (a+b arctanh(c x))^p.m"

Problem 4: Result more than twice size of optimal antiderivative.

$$\int (d + c\, d\, x)\, (a + b\, \text{ArcTanh}[c\, x])\, dx$$

Optimal (type 3, 44 leaves, 4 steps):

$$\frac{b\, d\, x}{2} + \frac{d\, (1 + c\, x)^2\, (a + b\, \text{ArcTanh}[c\, x])}{2\, c} + \frac{b\, d\, \text{Log}[1 - c\, x]}{c}$$

Result (type 3, 95 leaves):

$$a\, d\, x + \frac{b\, d\, x}{2} + \frac{1}{2}\, a\, c\, d\, x^2 + b\, d\, x\, \text{ArcTanh}[c\, x] + \frac{1}{2}\, b\, c\, d\, x^2\, \text{ArcTanh}[c\, x] + \frac{b\, d\, \text{Log}[1 - c\, x]}{4\, c} - \frac{b\, d\, \text{Log}[1 + c\, x]}{4\, c} + \frac{b\, d\, \text{Log}\left[1 - c^2\, x^2\right]}{2\, c}$$

Problem 72: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{(d + c\, d\, x)\, (a + b\, \text{ArcTanh}[c\, x])^2}{x}\, dx$$

Optimal (type 4, 191 leaves, 13 steps):

$$d\, (a + b\, \text{ArcTanh}[c\, x])^2 + c\, d\, x\, (a + b\, \text{ArcTanh}[c\, x])^2 + 2\, d\, (a + b\, \text{ArcTanh}[c\, x])^2\, \text{ArcTanh}\left[1 - \frac{2}{1 - c\, x}\right] - 2\, b\, d\, (a + b\, \text{ArcTanh}[c\, x])\, \text{Log}\left[\frac{2}{1 - c\, x}\right] - b^2\, d\, \text{PolyLog}\left[2, 1 - \frac{2}{1 - c\, x}\right] - b\, d\, (a + b\, \text{ArcTanh}[c\, x])\, \text{PolyLog}\left[2, 1 - \frac{2}{1 - c\, x}\right] + b\, d\, (a + b\, \text{ArcTanh}[c\, x])\, \text{PolyLog}\left[2, -1 + \frac{2}{1 - c\, x}\right] + \frac{1}{2}\, b^2\, d\, \text{PolyLog}\left[3, 1 - \frac{2}{1 - c\, x}\right] - \frac{1}{2}\, b^2\, d\, \text{PolyLog}\left[3, -1 + \frac{2}{1 - c\, x}\right]$$

Result (type 4, 228 leaves):

$$d\left(a^2 c x+a^2 \text{Log}[c x]+a b\left(2 c x \text{ArcTanh}[c x]+\text{Log}\left[1-c^2 x^2\right]\right)+b^2\left(\text{ArcTanh}[c x]\left((-1+c x)\text{ArcTanh}[c x]-2 \text{Log}\left[1+e^{-2 \text{ArcTanh}[c x]}\right]\right)+\text{PolyLog}\left[2,-e^{-2 \text{ArcTanh}[c x]}\right]\right)+a b(-\text{PolyLog}[2,-c x]+\text{PolyLog}[2,c x])+b^2\left(\frac{i \pi^3}{24}-\frac{2}{3}\text{ArcTanh}[c x]^3-\text{ArcTanh}[c x]^2 \text{Log}\left[1+e^{-2 \text{ArcTanh}[c x]}\right]+\text{ArcTanh}[c x]^2 \text{Log}\left[1-e^{2 \text{ArcTanh}[c x]}\right]+\text{ArcTanh}[c x]\text{PolyLog}\left[2,-e^{-2 \text{ArcTanh}[c x]}\right]+\text{ArcTanh}[c x]\text{PolyLog}\left[2,e^{2 \text{ArcTanh}[c x]}\right]+\frac{1}{2}\text{PolyLog}\left[3,-e^{-2 \text{ArcTanh}[c x]}\right]-\frac{1}{2}\text{PolyLog}\left[3,e^{2 \text{ArcTanh}[c x]}\right]\right)\right)$$

Problem 73: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{(d+c d x)(a+b \text{ArcTanh}[c x])^2}{x^2}\,dx$$

Optimal (type 4, 201 leaves, 12 steps):

$$c d (a+b \text{ArcTanh}[c x])^2-\frac{d (a+b \text{ArcTanh}[c x])^2}{x}+2 c d (a+b \text{ArcTanh}[c x])^2 \text{ArcTanh}\left[1-\frac{2}{1-c x}\right]+2 b c d (a+b \text{ArcTanh}[c x]) \text{Log}\left[2-\frac{2}{1+c x}\right]-b c d (a+b \text{ArcTanh}[c x]) \text{PolyLog}\left[2,1-\frac{2}{1-c x}\right]+b c d (a+b \text{ArcTanh}[c x]) \text{PolyLog}\left[2,-1+\frac{2}{1-c x}\right]-b^2 c d \text{PolyLog}\left[2,-1+\frac{2}{1+c x}\right]+\frac{1}{2} b^2 c d \text{PolyLog}\left[3,1-\frac{2}{1-c x}\right]-\frac{1}{2} b^2 c d \text{PolyLog}\left[3,-1+\frac{2}{1-c x}\right]$$

Result (type 4, 249 leaves):

$$-\frac{1}{x} d\left(a^2-a^2 c x \text{Log}[x]+a b\left(2 \text{ArcTanh}[c x]+c x\left(-2 \text{Log}[c x]+\text{Log}\left[1-c^2 x^2\right]\right)\right)+b^2\left(\text{ArcTanh}[c x]\left((1-c x)\text{ArcTanh}[c x]-2 c x \text{Log}\left[1-e^{-2 \text{ArcTanh}[c x]}\right]\right)+c x \text{PolyLog}\left[2,e^{-2 \text{ArcTanh}[c x]}\right]\right)+a b c x(\text{PolyLog}[2,-c x]-\text{PolyLog}[2,c x])-b^2 c x\left(\frac{i \pi^3}{24}-\frac{2}{3}\text{ArcTanh}[c x]^3-\text{ArcTanh}[c x]^2 \text{Log}\left[1+e^{-2 \text{ArcTanh}[c x]}\right]+\text{ArcTanh}[c x]^2 \text{Log}\left[1-e^{2 \text{ArcTanh}[c x]}\right]+\text{ArcTanh}[c x]\text{PolyLog}\left[2,-e^{-2 \text{ArcTanh}[c x]}\right]+\text{ArcTanh}[c x]\text{PolyLog}\left[2,e^{2 \text{ArcTanh}[c x]}\right]+\frac{1}{2}\text{PolyLog}\left[3,-e^{-2 \text{ArcTanh}[c x]}\right]-\frac{1}{2}\text{PolyLog}\left[3,e^{2 \text{ArcTanh}[c x]}\right]\right)\right)$$

Problem 80: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{(d+c d x)^2 (a+b \text{ArcTanh}[c x])^2}{x}\,dx$$

Optimal (type 4, 278 leaves, 19 steps):

$$a\,b\,c\,d^2\,x+b^2\,c\,d^2\,x\,\text{ArcTanh}[c\,x]+\frac{3}{2}\,d^2\,(a+b\,\text{ArcTanh}[c\,x])^2+2\,c\,d^2\,x\,(a+b\,\text{ArcTanh}[c\,x])^2+\frac{1}{2}\,c^2\,d^2\,x^2\,(a+b\,\text{ArcTanh}[c\,x])^2+2\,d^2\,(a+b\,\text{ArcTanh}[c\,x])^2\,\text{ArcTanh}\left[1-\frac{2}{1-c\,x}\right]-4\,b\,d^2\,(a+b\,\text{ArcTanh}[c\,x])\,\text{Log}\left[\frac{2}{1-c\,x}\right]+\frac{1}{2}\,b^2\,d^2\,\text{Log}\left[1-c^2\,x^2\right]-2\,b^2\,d^2\,\text{PolyLog}\left[2,1-\frac{2}{1-c\,x}\right]-b\,d^2\,(a+b\,\text{ArcTanh}[c\,x])\,\text{PolyLog}\left[2,1-\frac{2}{1-c\,x}\right]+b\,d^2\,(a+b\,\text{ArcTanh}[c\,x])\,\text{PolyLog}\left[2,-1+\frac{2}{1-c\,x}\right]+\frac{1}{2}\,b^2\,d^2\,\text{PolyLog}\left[3,1-\frac{2}{1-c\,x}\right]-\frac{1}{2}\,b^2\,d^2\,\text{PolyLog}\left[3,-1+\frac{2}{1-c\,x}\right]$$

Result (type 4, 324 leaves):

$$\frac{1}{2}\,d^2\left(4\,a^2\,c\,x+a^2\,c^2\,x^2+2\,a^2\,\text{Log}[c\,x]+a\,b\,\left(2\,c\,x+2\,c^2\,x^2\,\text{ArcTanh}[c\,x]+\text{Log}[1-c\,x]-\text{Log}[1+c\,x]\right)+4\,a\,b\,\left(2\,c\,x\,\text{ArcTanh}[c\,x]+\text{Log}\left[1-c^2\,x^2\right]\right)+b^2\,\left(2\,c\,x\,\text{ArcTanh}[c\,x]+\left(-1+c^2\,x^2\right)\,\text{ArcTanh}[c\,x]^2+\text{Log}\left[1-c^2\,x^2\right]\right)+4\,b^2\,\left(\text{ArcTanh}[c\,x]\,\left(\left(-1+c\,x\right)\,\text{ArcTanh}[c\,x]-2\,\text{Log}\left[1+e^{-2\,\text{ArcTanh}[c\,x]}\right]\right)+\text{PolyLog}\left[2,-e^{-2\,\text{ArcTanh}[c\,x]}\right]\right)+2\,a\,b\,\left(-\text{PolyLog}[2,-c\,x]+\text{PolyLog}[2,c\,x]\right)+2\,b^2\,\left(\frac{i\,\pi^3}{24}-\frac{2}{3}\,\text{ArcTanh}[c\,x]^3-\text{ArcTanh}[c\,x]^2\,\text{Log}\left[1+e^{-2\,\text{ArcTanh}[c\,x]}\right]+\text{ArcTanh}[c\,x]^2\,\text{Log}\left[1-e^{2\,\text{ArcTanh}[c\,x]}\right]+\text{ArcTanh}[c\,x]\,\text{PolyLog}\left[2,-e^{-2\,\text{ArcTanh}[c\,x]}\right]+\text{ArcTanh}[c\,x]\,\text{PolyLog}\left[2,e^{2\,\text{ArcTanh}[c\,x]}\right]+\frac{1}{2}\,\text{PolyLog}\left[3,-e^{-2\,\text{ArcTanh}[c\,x]}\right]-\frac{1}{2}\,\text{PolyLog}\left[3,e^{2\,\text{ArcTanh}[c\,x]}\right]\right)\right)$$

Problem 81: Result unnecessarily involves imaginary or complex numbers.

$$\int\frac{(d+c\,d\,x)^2\,(a+b\,\text{ArcTanh}[c\,x])^2}{x^2}\,dx$$

Optimal (type 4, 283 leaves, 17 steps):

$$2\,c\,d^2\,(a+b\,\text{ArcTanh}[c\,x])^2-\frac{d^2\,(a+b\,\text{ArcTanh}[c\,x])^2}{x}+c^2\,d^2\,x\,(a+b\,\text{ArcTanh}[c\,x])^2+4\,c\,d^2\,(a+b\,\text{ArcTanh}[c\,x])^2\,\text{ArcTanh}\left[1-\frac{2}{1-c\,x}\right]-2\,b\,c\,d^2\,(a+b\,\text{ArcTanh}[c\,x])\,\text{Log}\left[\frac{2}{1-c\,x}\right]+2\,b\,c\,d^2\,(a+b\,\text{ArcTanh}[c\,x])\,\text{Log}\left[2-\frac{2}{1+c\,x}\right]-b^2\,c\,d^2\,\text{PolyLog}\left[2,1-\frac{2}{1-c\,x}\right]-2\,b\,c\,d^2\,(a+b\,\text{ArcTanh}[c\,x])\,\text{PolyLog}\left[2,1-\frac{2}{1-c\,x}\right]+2\,b\,c\,d^2\,(a+b\,\text{ArcTanh}[c\,x])\,\text{PolyLog}\left[2,-1+\frac{2}{1-c\,x}\right]-b^2\,c\,d^2\,\text{PolyLog}\left[2,-1+\frac{2}{1+c\,x}\right]+b^2\,c\,d^2\,\text{PolyLog}\left[3,1-\frac{2}{1-c\,x}\right]-b^2\,c\,d^2\,\text{PolyLog}\left[3,-1+\frac{2}{1-c\,x}\right]$$

Result (type 4, 341 leaves):

$$\frac{1}{12\,x} d^2 \left(-12 a^2 + i b^2 c \pi^3 x + 12 a^2 c^2 x^2 - 24 a b \text{ArcTanh}[c x] + 24 a b c^2 x^2 \text{ArcTanh}[c x] - 12 b^2 \text{ArcTanh}[c x]^2 + 12 b^2 c^2 x^2 \text{ArcTanh}[c x]^2 - 16 b^2 c x \text{ArcTanh}[c x]^3 + 24 b^2 c x \text{ArcTanh}[c x] \text{Log}\left[1 - e^{-2 \text{ArcTanh}[c x]}\right] - 24 b^2 c x \text{ArcTanh}[c x] \text{Log}\left[1 + e^{-2 \text{ArcTanh}[c x]}\right] - 24 b^2 c x \text{ArcTanh}[c x]^2 \text{Log}\left[1 + e^{-2 \text{ArcTanh}[c x]}\right] + 24 b^2 c x \text{ArcTanh}[c x]^2 \text{Log}\left[1 - e^{2 \text{ArcTanh}[c x]}\right] + 24 a^2 c x \text{Log}[x] + 24 a b c x \text{Log}[c x] + 12 b^2 c x (1 + 2 \text{ArcTanh}[c x]) \text{PolyLog}\left[2, -e^{-2 \text{ArcTanh}[c x]}\right] - 12 b^2 c x \text{PolyLog}\left[2, e^{-2 \text{ArcTanh}[c x]}\right] + 24 b^2 c x \text{ArcTanh}[c x] \text{PolyLog}\left[2, e^{2 \text{ArcTanh}[c x]}\right] - 24 a b c x \text{PolyLog}[2, -c x] + 24 a b c x \text{PolyLog}[2, c x] + 12 b^2 c x \text{PolyLog}\left[3, -e^{-2 \text{ArcTanh}[c x]}\right] - 12 b^2 c x \text{PolyLog}\left[3, e^{2 \text{ArcTanh}[c x]}\right]\right)$$

Problem 82: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{(d + c d x)^2 (a + b \text{ArcTanh}[c x])^2}{x^3} \, dx$$

Optimal (type 4, 313 leaves, 20 steps):

$$-\frac{b c d^2 (a + b \text{ArcTanh}[c x])}{x} + \frac{5}{2} c^2 d^2 (a + b \text{ArcTanh}[c x])^2 - \frac{d^2 (a + b \text{ArcTanh}[c x])^2}{2 x^2} - \frac{2 c d^2 (a + b \text{ArcTanh}[c x])^2}{x} + 2 c^2 d^2 (a + b \text{ArcTanh}[c x])^2 \text{ArcTanh}\left[1 - \frac{2}{1 - c x}\right] + b^2 c^2 d^2 \text{Log}[x] - \frac{1}{2} b^2 c^2 d^2 \text{Log}\left[1 - c^2 x^2\right] + 4 b c^2 d^2 (a + b \text{ArcTanh}[c x]) \text{Log}\left[2 - \frac{2}{1 + c x}\right] - b c^2 d^2 (a + b \text{ArcTanh}[c x]) \text{PolyLog}\left[2, 1 - \frac{2}{1 - c x}\right] + b c^2 d^2 (a + b \text{ArcTanh}[c x]) \text{PolyLog}\left[2, -1 + \frac{2}{1 - c x}\right] - 2 b^2 c^2 d^2 \text{PolyLog}\left[2, -1 + \frac{2}{1 + c x}\right] + \frac{1}{2} b^2 c^2 d^2 \text{PolyLog}\left[3, 1 - \frac{2}{1 - c x}\right] - \frac{1}{2} b^2 c^2 d^2 \text{PolyLog}\left[3, -1 + \frac{2}{1 - c x}\right]$$

Result (type 4, 370 leaves):

$$-\frac{1}{2 x^2} d^2 \left(a^2 + 4 a^2 c x - 2 a^2 c^2 x^2 \text{Log}[x] + a b \left(2 \text{ArcTanh}[c x] + c x (2 + c x \text{Log}[1 - c x] - c x \text{Log}[1 + c x])\right) + b^2 \left(2 c x \text{ArcTanh}[c x] + (1 - c^2 x^2) \text{ArcTanh}[c x]^2 - 2 c^2 x^2 \text{Log}\left[\frac{c x}{\sqrt{1 - c^2 x^2}}\right]\right) + 4 a b c x \left(2 \text{ArcTanh}[c x] + c x \left(-2 \text{Log}[c x] + \text{Log}\left[1 - c^2 x^2\right]\right)\right) + 4 b^2 c x \left(\text{ArcTanh}[c x] \left((1 - c x) \text{ArcTanh}[c x] - 2 c x \text{Log}\left[1 - e^{-2 \text{ArcTanh}[c x]}\right]\right) + c x \text{PolyLog}\left[2, e^{-2 \text{ArcTanh}[c x]}\right]\right) + 2 a b c^2 x^2 (\text{PolyLog}[2, -c x] - \text{PolyLog}[2, c x]) - 2 b^2 c^2 x^2 \left(\frac{i \pi^3}{24} - \frac{2}{3} \text{ArcTanh}[c x]^3 - \text{ArcTanh}[c x]^2 \text{Log}\left[1 + e^{-2 \text{ArcTanh}[c x]}\right] + \text{ArcTanh}[c x]^2 \text{Log}\left[1 - e^{2 \text{ArcTanh}[c x]}\right] + \text{ArcTanh}[c x] \text{PolyLog}\left[2, -e^{-2 \text{ArcTanh}[c x]}\right] + \text{ArcTanh}[c x] \text{PolyLog}\left[2, e^{2 \text{ArcTanh}[c x]}\right] + \frac{1}{2} \text{PolyLog}\left[3, -e^{-2 \text{ArcTanh}[c x]}\right] - \frac{1}{2} \text{PolyLog}\left[3, e^{2 \text{ArcTanh}[c x]}\right]\right)\right)$$

Problem 88: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{(d + c\,d\,x)^3\,(a + b\,\text{ArcTanh}[c\,x])^2}{x}\,dx$$

Optimal (type 4, 355 leaves, 28 steps):

$$3\,a\,b\,c\,d^3\,x + \frac{1}{3}\,b^2\,c\,d^3\,x - \frac{1}{3}\,b^2\,d^3\,\text{ArcTanh}[c\,x] + 3\,b^2\,c\,d^3\,x\,\text{ArcTanh}[c\,x] + \frac{1}{3}\,b\,c^2\,d^3\,x^2\,(a + b\,\text{ArcTanh}[c\,x]) + \frac{11}{6}\,d^3\,(a + b\,\text{ArcTanh}[c\,x])^2 + 3\,c\,d^3\,x\,(a + b\,\text{ArcTanh}[c\,x])^2 + \frac{3}{2}\,c^2\,d^3\,x^2\,(a + b\,\text{ArcTanh}[c\,x])^2 + \frac{1}{3}\,c^3\,d^3\,x^3\,(a + b\,\text{ArcTanh}[c\,x])^2 + 2\,d^3\,(a + b\,\text{ArcTanh}[c\,x])^2\,\text{ArcTanh}\left[1 - \frac{2}{1 - c\,x}\right] - \frac{20}{3}\,b\,d^3\,(a + b\,\text{ArcTanh}[c\,x])\,\text{Log}\left[\frac{2}{1 - c\,x}\right] + \frac{3}{2}\,b^2\,d^3\,\text{Log}\left[1 - c^2\,x^2\right] - \frac{10}{3}\,b^2\,d^3\,\text{PolyLog}\left[2, 1 - \frac{2}{1 - c\,x}\right] - b\,d^3\,(a + b\,\text{ArcTanh}[c\,x])\,\text{PolyLog}\left[2, 1 - \frac{2}{1 - c\,x}\right] + b\,d^3\,(a + b\,\text{ArcTanh}[c\,x])\,\text{PolyLog}\left[2, -1 + \frac{2}{1 - c\,x}\right] + \frac{1}{2}\,b^2\,d^3\,\text{PolyLog}\left[3, 1 - \frac{2}{1 - c\,x}\right] - \frac{1}{2}\,b^2\,d^3\,\text{PolyLog}\left[3, -1 + \frac{2}{1 - c\,x}\right]$$

Result (type 4, 448 leaves):

$$\frac{1}{24}\,d^3\,\Big(i\,b^2\,\pi^3 + 72\,a^2\,c\,x + 72\,a\,b\,c\,x + 8\,b^2\,c\,x + 36\,a^2\,c^2\,x^2 + 8\,a\,b\,c^2\,x^2 + 8\,a^2\,c^3\,x^3 - 8\,b^2\,\text{ArcTanh}[c\,x] + 144\,a\,b\,c\,x\,\text{ArcTanh}[c\,x] + 72\,b^2\,c\,x\,\text{ArcTanh}[c\,x] + 72\,a\,b\,c^2\,x^2\,\text{ArcTanh}[c\,x] + 8\,b^2\,c^2\,x^2\,\text{ArcTanh}[c\,x] + 16\,a\,b\,c^3\,x^3\,\text{ArcTanh}[c\,x] - 116\,b^2\,\text{ArcTanh}[c\,x]^2 + 72\,b^2\,c\,x\,\text{ArcTanh}[c\,x]^2 + 36\,b^2\,c^2\,x^2\,\text{ArcTanh}[c\,x]^2 + 8\,b^2\,c^3\,x^3\,\text{ArcTanh}[c\,x]^2 - 16\,b^2\,\text{ArcTanh}[c\,x]^3 - 160\,b^2\,\text{ArcTanh}[c\,x]\,\text{Log}\left[1 + e^{-2\,\text{ArcTanh}[c\,x]}\right] - 24\,b^2\,\text{ArcTanh}[c\,x]^2\,\text{Log}\left[1 + e^{-2\,\text{ArcTanh}[c\,x]}\right] + 24\,b^2\,\text{ArcTanh}[c\,x]^2\,\text{Log}\left[1 - e^{2\,\text{ArcTanh}[c\,x]}\right] + 24\,a^2\,\text{Log}[c\,x] + 36\,a\,b\,\text{Log}[1 - c\,x] - 36\,a\,b\,\text{Log}[1 + c\,x] + 72\,a\,b\,\text{Log}\left[1 - c^2\,x^2\right] + 36\,b^2\,\text{Log}\left[1 - c^2\,x^2\right] + 8\,a\,b\,\text{Log}\left[-1 + c^2\,x^2\right] + 8\,b^2\,(10 + 3\,\text{ArcTanh}[c\,x])\,\text{PolyLog}\left[2, -e^{-2\,\text{ArcTanh}[c\,x]}\right] + 24\,b^2\,\text{ArcTanh}[c\,x]\,\text{PolyLog}\left[2, e^{2\,\text{ArcTanh}[c\,x]}\right] - 24\,a\,b\,\text{PolyLog}[2, -c\,x] + 24\,a\,b\,\text{PolyLog}[2, c\,x] + 12\,b^2\,\text{PolyLog}\left[3, -e^{-2\,\text{ArcTanh}[c\,x]}\right] - 12\,b^2\,\text{PolyLog}\left[3, e^{2\,\text{ArcTanh}[c\,x]}\right]\Big)$$

Problem 89: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{(d + c\,d\,x)^3\,(a + b\,\text{ArcTanh}[c\,x])^2}{x^2}\,dx$$

Optimal (type 4, 361 leaves, 23 steps):

$$a\,b\,c^2\,d^3\,x+b^2\,c^2\,d^3\,x\,\text{ArcTanh}[c\,x]+\frac{7}{2}\,c\,d^3\,\left(a+b\,\text{ArcTanh}[c\,x]\right)^2-\frac{d^3\,\left(a+b\,\text{ArcTanh}[c\,x]\right)^2}{x}+3\,c^2\,d^3\,x\,\left(a+b\,\text{ArcTanh}[c\,x]\right)^2+\frac{1}{2}\,c^3\,d^3\,x^2\,\left(a+b\,\text{ArcTanh}[c\,x]\right)^2+6\,c\,d^3\,\left(a+b\,\text{ArcTanh}[c\,x]\right)^2\,\text{ArcTanh}\left[1-\frac{2}{1-c\,x}\right]-6\,b\,c\,d^3\,\left(a+b\,\text{ArcTanh}[c\,x]\right)\,\text{Log}\left[\frac{2}{1-c\,x}\right]+\frac{1}{2}\,b^2\,c\,d^3\,\text{Log}\left[1-c^2\,x^2\right]+2\,b\,c\,d^3\,\left(a+b\,\text{ArcTanh}[c\,x]\right)\,\text{Log}\left[2-\frac{2}{1+c\,x}\right]-3\,b^2\,c\,d^3\,\text{PolyLog}\left[2,\,1-\frac{2}{1-c\,x}\right]-3\,b\,c\,d^3\,\left(a+b\,\text{ArcTanh}[c\,x]\right)\,\text{PolyLog}\left[2,\,1-\frac{2}{1-c\,x}\right]+3\,b\,c\,d^3\,\left(a+b\,\text{ArcTanh}[c\,x]\right)\,\text{PolyLog}\left[2,\,-1+\frac{2}{1-c\,x}\right]-b^2\,c\,d^3\,\text{PolyLog}\left[2,\,-1+\frac{2}{1+c\,x}\right]+\frac{3}{2}\,b^2\,c\,d^3\,\text{PolyLog}\left[3,\,1-\frac{2}{1-c\,x}\right]-\frac{3}{2}\,b^2\,c\,d^3\,\text{PolyLog}\left[3,\,-1+\frac{2}{1-c\,x}\right]$$

Result (type 4, 479 leaves):

$$\frac{1}{8\,x}\,d^3\,\Big(-8\,a^2+i\,b^2\,c\,\pi^3\,x+24\,a^2\,c^2\,x^2+8\,a\,b\,c^2\,x^2+4\,a^2\,c^3\,x^3-16\,a\,b\,\text{ArcTanh}[c\,x]+48\,a\,b\,c^2\,x^2\,\text{ArcTanh}[c\,x]+8\,b^2\,c^2\,x^2\,\text{ArcTanh}[c\,x]+8\,a\,b\,c^3\,x^3\,\text{ArcTanh}[c\,x]-8\,b^2\,\text{ArcTanh}[c\,x]^2-20\,b^2\,c\,x\,\text{ArcTanh}[c\,x]^2+24\,b^2\,c^2\,x^2\,\text{ArcTanh}[c\,x]^2+4\,b^2\,c^3\,x^3\,\text{ArcTanh}[c\,x]^2-16\,b^2\,c\,x\,\text{ArcTanh}[c\,x]^3+16\,b^2\,c\,x\,\text{ArcTanh}[c\,x]\,\text{Log}\left[1-e^{-2\,\text{ArcTanh}[c\,x]}\right]-48\,b^2\,c\,x\,\text{ArcTanh}[c\,x]\,\text{Log}\left[1+e^{-2\,\text{ArcTanh}[c\,x]}\right]-24\,b^2\,c\,x\,\text{ArcTanh}[c\,x]^2\,\text{Log}\left[1+e^{-2\,\text{ArcTanh}[c\,x]}\right]+24\,b^2\,c\,x\,\text{ArcTanh}[c\,x]^2\,\text{Log}\left[1-e^{2\,\text{ArcTanh}[c\,x]}\right]+24\,a^2\,c\,x\,\text{Log}[x]+16\,a\,b\,c\,x\,\text{Log}[c\,x]+4\,a\,b\,c\,x\,\text{Log}[1-c\,x]-4\,a\,b\,c\,x\,\text{Log}[1+c\,x]+16\,a\,b\,c\,x\,\text{Log}\left[1-c^2\,x^2\right]+4\,b^2\,c\,x\,\text{Log}\left[1-c^2\,x^2\right]+24\,b^2\,c\,x\,\left(1+\text{ArcTanh}[c\,x]\right)\,\text{PolyLog}\left[2,\,-e^{-2\,\text{ArcTanh}[c\,x]}\right]-8\,b^2\,c\,x\,\text{PolyLog}\left[2,\,e^{-2\,\text{ArcTanh}[c\,x]}\right]+24\,b^2\,c\,x\,\text{ArcTanh}[c\,x]\,\text{PolyLog}\left[2,\,e^{2\,\text{ArcTanh}[c\,x]}\right]-24\,a\,b\,c\,x\,\text{PolyLog}[2,\,-c\,x]+24\,a\,b\,c\,x\,\text{PolyLog}[2,\,c\,x]+12\,b^2\,c\,x\,\text{PolyLog}\left[3,\,-e^{-2\,\text{ArcTanh}[c\,x]}\right]-12\,b^2\,c\,x\,\text{PolyLog}\left[3,\,e^{2\,\text{ArcTanh}[c\,x]}\right]\Big)$$

Problem 90: Result unnecessarily involves imaginary or complex numbers.

$$\int\frac{(d+c\,d\,x)^3\,\left(a+b\,\text{ArcTanh}[c\,x]\right)^2}{x^3}\,dx$$

Optimal (type 4, 385 leaves, 25 steps):

$$-\frac{b\,c\,d^3\,(a+b\,\text{ArcTanh}[c\,x])}{x}+\frac{9}{2}c^2\,d^3\,(a+b\,\text{ArcTanh}[c\,x])^2-\frac{d^3\,(a+b\,\text{ArcTanh}[c\,x])^2}{2\,x^2}-\frac{3\,c\,d^3\,(a+b\,\text{ArcTanh}[c\,x])^2}{x}+c^3\,d^3\,x\,(a+b\,\text{ArcTanh}[c\,x])^2+6\,c^2\,d^3\,(a+b\,\text{ArcTanh}[c\,x])^2\,\text{ArcTanh}\left[1-\frac{2}{1-c\,x}\right]+b^2\,c^2\,d^3\,\text{Log}[x]-2\,b\,c^2\,d^3\,(a+b\,\text{ArcTanh}[c\,x])\,\text{Log}\left[\frac{2}{1-c\,x}\right]-\frac{1}{2}b^2\,c^2\,d^3\,\text{Log}\left[1-c^2\,x^2\right]+6\,b\,c^2\,d^3\,(a+b\,\text{ArcTanh}[c\,x])\,\text{Log}\left[2-\frac{2}{1+c\,x}\right]-b^2\,c^2\,d^3\,\text{PolyLog}\left[2,1-\frac{2}{1-c\,x}\right]-3\,b\,c^2\,d^3\,(a+b\,\text{ArcTanh}[c\,x])\,\text{PolyLog}\left[2,1-\frac{2}{1-c\,x}\right]+3\,b\,c^2\,d^3\,(a+b\,\text{ArcTanh}[c\,x])\,\text{PolyLog}\left[2,-1+\frac{2}{1-c\,x}\right]-3\,b^2\,c^2\,d^3\,\text{PolyLog}\left[2,-1+\frac{2}{1+c\,x}\right]+\frac{3}{2}b^2\,c^2\,d^3\,\text{PolyLog}\left[3,1-\frac{2}{1-c\,x}\right]-\frac{3}{2}b^2\,c^2\,d^3\,\text{PolyLog}\left[3,-1+\frac{2}{1-c\,x}\right]$$

Result (type 4, 461 leaves) :

$$\frac{1}{2}d^3\left(-\frac{a^2}{x^2}-\frac{6\,a^2\,c}{x}+2\,a^2\,c^3\,x+6\,a^2\,c^2\,\text{Log}[x]-\frac{a\,b\,(2\,\text{ArcTanh}[c\,x]+c\,x\,(2+c\,x\,\text{Log}[1-c\,x]-c\,x\,\text{Log}[1+c\,x]))}{x^2}+\frac{1}{x^2}b^2\left(-2\,c\,x\,\text{ArcTanh}[c\,x]+\left(-1+c^2\,x^2\right)\text{ArcTanh}[c\,x]^2+2\,c^2\,x^2\,\text{Log}\left[\frac{c\,x}{\sqrt{1-c^2\,x^2}}\right]\right)+2\,a\,b\,c^2\,\left(2\,c\,x\,\text{ArcTanh}[c\,x]+\text{Log}\left[1-c^2\,x^2\right]\right)-\frac{6\,a\,b\,c\,\left(2\,\text{ArcTanh}[c\,x]+c\,x\,\left(-2\,\text{Log}[c\,x]+\text{Log}\left[1-c^2\,x^2\right]\right)\right)}{x}+2\,b^2\,c^2\,\left(\text{ArcTanh}[c\,x]\,\left((-1+c\,x)\,\text{ArcTanh}[c\,x]-2\,\text{Log}\left[1+e^{-2\,\text{ArcTanh}[c\,x]}\right]\right)+\text{PolyLog}\left[2,-e^{-2\,\text{ArcTanh}[c\,x]}\right]\right)+\frac{1}{x}6\,b^2\,c\,\left(\text{ArcTanh}[c\,x]\,\left((-1+c\,x)\,\text{ArcTanh}[c\,x]+2\,c\,x\,\text{Log}\left[1-e^{-2\,\text{ArcTanh}[c\,x]}\right]\right)-c\,x\,\text{PolyLog}\left[2,e^{-2\,\text{ArcTanh}[c\,x]}\right]\right)-6\,a\,b\,c^2\,(\text{PolyLog}[2,-c\,x]-\text{PolyLog}[2,c\,x])+6\,b^2\,c^2\left(\frac{i\,\pi^3}{24}-\frac{2}{3}\text{ArcTanh}[c\,x]^3-\text{ArcTanh}[c\,x]^2\,\text{Log}\left[1+e^{-2\,\text{ArcTanh}[c\,x]}\right]+\text{ArcTanh}[c\,x]^2\,\text{Log}\left[1-e^{2\,\text{ArcTanh}[c\,x]}\right]+\text{ArcTanh}[c\,x]\,\text{PolyLog}\left[2,-e^{-2\,\text{ArcTanh}[c\,x]}\right]+\text{ArcTanh}[c\,x]\,\text{PolyLog}\left[2,e^{2\,\text{ArcTanh}[c\,x]}\right]+\frac{1}{2}\text{PolyLog}\left[3,-e^{-2\,\text{ArcTanh}[c\,x]}\right]-\frac{1}{2}\text{PolyLog}\left[3,e^{2\,\text{ArcTanh}[c\,x]}\right]\right)\right)$$

Problem 91: Result unnecessarily involves imaginary or complex numbers.

$$\int\frac{(d+c\,d\,x)^3\,(a+b\,\text{ArcTanh}[c\,x])^2}{x^4}\,dx$$

Optimal (type 4, 396 leaves, 28 steps) :

$$-\frac{b^2 c^2 d^3}{3 x}+\frac{1}{3} b^2 c^3 d^3 \text{ArcTanh}[c x]-\frac{b c d^3 (a+b \text{ArcTanh}[c x])}{3 x^2}-\frac{3 b c^2 d^3 (a+b \text{ArcTanh}[c x])}{x}+\frac{29}{6} c^3 d^3 (a+b \text{ArcTanh}[c x])^2-\frac{d^3 (a+b \text{ArcTanh}[c x])^2}{3 x^3}-\frac{3 c d^3 (a+b \text{ArcTanh}[c x])^2}{2 x^2}-\frac{3 c^2 d^3 (a+b \text{ArcTanh}[c x])^2}{x}+2 c^3 d^3 (a+b \text{ArcTanh}[c x])^2 \text{ArcTanh}\left[1-\frac{2}{1-c x}\right]+3 b^2 c^3 d^3 \text{Log}[x]-\frac{3}{2} b^2 c^3 d^3 \text{Log}\left[1-c^2 x^2\right]+\frac{20}{3} b c^3 d^3 (a+b \text{ArcTanh}[c x]) \text{Log}\left[2-\frac{2}{1+c x}\right]-b c^3 d^3 (a+b \text{ArcTanh}[c x]) \text{PolyLog}\left[2,1-\frac{2}{1-c x}\right]+b c^3 d^3 (a+b \text{ArcTanh}[c x]) \text{PolyLog}\left[2,-1+\frac{2}{1-c x}\right]-\frac{10}{3} b^2 c^3 d^3 \text{PolyLog}\left[2,-1+\frac{2}{1+c x}\right]+\frac{1}{2} b^2 c^3 d^3 \text{PolyLog}\left[3,1-\frac{2}{1-c x}\right]-\frac{1}{2} b^2 c^3 d^3 \text{PolyLog}\left[3,-1+\frac{2}{1-c x}\right]$$

Result (type 4, 569 leaves):

$$\frac{1}{24 x^3} d^3 \Big(-8 a^2-36 a^2 c x-8 a b c x-72 a^2 c^2 x^2-72 a b c^2 x^2-8 b^2 c^2 x^2+i b^2 c^3 \pi^3 x^3-16 a b \text{ArcTanh}[c x]-72 a b c x \text{ArcTanh}[c x]-8 b^2 c x \text{ArcTanh}[c x]-144 a b c^2 x^2 \text{ArcTanh}[c x]-72 b^2 c^2 x^2 \text{ArcTanh}[c x]+8 b^2 c^3 x^3 \text{ArcTanh}[c x]-8 b^2 \text{ArcTanh}[c x]^2-36 b^2 c x \text{ArcTanh}[c x]^2-72 b^2 c^2 x^2 \text{ArcTanh}[c x]^2+116 b^2 c^3 x^3 \text{ArcTanh}[c x]^2-16 b^2 c^3 x^3 \text{ArcTanh}[c x]^3+160 b^2 c^3 x^3 \text{ArcTanh}[c x] \text{Log}\left[1-e^{-2 \text{ArcTanh}[c x]}\right]-24 b^2 c^3 x^3 \text{ArcTanh}[c x]^2 \text{Log}\left[1+e^{-2 \text{ArcTanh}[c x]}\right]+24 b^2 c^3 x^3 \text{ArcTanh}[c x]^2 \text{Log}\left[1-e^{2 \text{ArcTanh}[c x]}\right]+24 a^2 c^3 x^3 \text{Log}[x]+160 a b c^3 x^3 \text{Log}[c x]-36 a b c^3 x^3 \text{Log}[1-c x]+36 a b c^3 x^3 \text{Log}[1+c x]+72 b^2 c^3 x^3 \text{Log}\left[\frac{c x}{\sqrt{1-c^2 x^2}}\right]-80 a b c^3 x^3 \text{Log}\left[1-c^2 x^2\right]+24 b^2 c^3 x^3 \text{ArcTanh}[c x] \text{PolyLog}\left[2,-e^{-2 \text{ArcTanh}[c x]}\right]-80 b^2 c^3 x^3 \text{PolyLog}\left[2,e^{-2 \text{ArcTanh}[c x]}\right]+24 b^2 c^3 x^3 \text{ArcTanh}[c x] \text{PolyLog}\left[2,e^{2 \text{ArcTanh}[c x]}\right]-24 a b c^3 x^3 \text{PolyLog}[2,-c x]+24 a b c^3 x^3 \text{PolyLog}[2,c x]+12 b^2 c^3 x^3 \text{PolyLog}\left[3,-e^{-2 \text{ArcTanh}[c x]}\right]-12 b^2 c^3 x^3 \text{PolyLog}\left[3,e^{2 \text{ArcTanh}[c x]}\right]\Big)$$

Problem 99: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{(a+b \text{ArcTanh}[c x])^2}{x (d+c d x)} \, dx$$

Optimal (type 4, 77 leaves, 3 steps):

$$\frac{(a+b \text{ArcTanh}[c x])^2 \text{Log}\left[2-\frac{2}{1+c x}\right]}{d}-\frac{b (a+b \text{ArcTanh}[c x]) \text{PolyLog}\left[2,-1+\frac{2}{1+c x}\right]}{d}-\frac{b^2 \text{PolyLog}\left[3,-1+\frac{2}{1+c x}\right]}{2 d}$$

Result (type 4, 132 leaves):

$$\frac{1}{d}\left(a^2 \text{Log}[c x]-a^2 \text{Log}[1+c x]+a b \left(2 \text{ArcTanh}[c x] \text{Log}\left[1-e^{-2 \text{ArcTanh}[c x]}\right]-\text{PolyLog}\left[2,e^{-2 \text{ArcTanh}[c x]}\right]\right)+b^2 \left(\frac{i \pi ^3}{24}-\frac{2}{3} \text{ArcTanh}[c x]^3+\text{ArcTanh}[c x]^2 \text{Log}\left[1-e^{2 \text{ArcTanh}[c x]}\right]+\text{ArcTanh}[c x] \text{PolyLog}\left[2,e^{2 \text{ArcTanh}[c x]}\right]-\frac{1}{2} \text{PolyLog}\left[3,e^{2 \text{ArcTanh}[c x]}\right]\right)\right)$$

Problem 100: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{(a+b \text{ArcTanh}[c x])^2}{x^2 (d+c d x)} \, dx$$

Optimal (type 4, 162 leaves, 8 steps):

$$\frac{c (a+b \text{ArcTanh}[c x])^2}{d}-\frac{(a+b \text{ArcTanh}[c x])^2}{d x}+\frac{2 b c (a+b \text{ArcTanh}[c x]) \text{Log}\left[2-\frac{2}{1+c x}\right]}{d}-\frac{c (a+b \text{ArcTanh}[c x])^2 \text{Log}\left[2-\frac{2}{1+c x}\right]}{d}-\frac{b^2 c \text{PolyLog}\left[2,-1+\frac{2}{1+c x}\right]}{d}+\frac{b c (a+b \text{ArcTanh}[c x]) \text{PolyLog}\left[2,-1+\frac{2}{1+c x}\right]}{d}+\frac{b^2 c \text{PolyLog}\left[3,-1+\frac{2}{1+c x}\right]}{2 d}$$

Result (type 4, 225 leaves):

$$\frac{1}{d}\left(-\frac{a^2}{x}-a^2 c \text{Log}[x]+a^2 c \text{Log}[1+c x]+\frac{1}{x}a b \left(-2 \text{ArcTanh}[c x] (1+c x \text{Log}\left[1-e^{-2 \text{ArcTanh}[c x]}\right])+2 c x \text{Log}\left[\frac{c x}{\sqrt{1-c^2 x^2}}\right]+c x \text{PolyLog}\left[2,e^{-2 \text{ArcTanh}[c x]}\right]\right)+b^2 c \left(-\frac{i \pi ^3}{24}+\text{ArcTanh}[c x]^2-\frac{\text{ArcTanh}[c x]^2}{c x}+\frac{2}{3} \text{ArcTanh}[c x]^3+2 \text{ArcTanh}[c x] \text{Log}\left[1-e^{-2 \text{ArcTanh}[c x]}\right]-\text{ArcTanh}[c x]^2 \text{Log}\left[1-e^{2 \text{ArcTanh}[c x]}\right]-\text{PolyLog}\left[2,e^{-2 \text{ArcTanh}[c x]}\right]-\text{ArcTanh}[c x] \text{PolyLog}\left[2,e^{2 \text{ArcTanh}[c x]}\right]+\frac{1}{2} \text{PolyLog}\left[3,e^{2 \text{ArcTanh}[c x]}\right]\right)\right)$$

Problem 101: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{(a+b \text{ArcTanh}[c x])^2}{x^3 (d+c d x)} \, dx$$

Optimal (type 4, 250 leaves, 17 steps):

$$-\frac{b\,c\,(a+b\,\text{ArcTanh}[c\,x])}{d\,x}-\frac{c^2\,(a+b\,\text{ArcTanh}[c\,x])^2}{2\,d}-\frac{(a+b\,\text{ArcTanh}[c\,x])^2}{2\,d\,x^2}+\frac{c\,(a+b\,\text{ArcTanh}[c\,x])^2}{d\,x}+\frac{b^2\,c^2\,\text{Log}[x]}{d}-\frac{b^2\,c^2\,\text{Log}[1-c^2\,x^2]}{2\,d}-\frac{2\,b\,c^2\,(a+b\,\text{ArcTanh}[c\,x])\,\text{Log}\left[2-\frac{2}{1+c\,x}\right]}{d}+\frac{c^2\,(a+b\,\text{ArcTanh}[c\,x])^2\,\text{Log}\left[2-\frac{2}{1+c\,x}\right]}{d}+\frac{b^2\,c^2\,\text{PolyLog}\left[2,-1+\frac{2}{1+c\,x}\right]}{d}-\frac{b\,c^2\,(a+b\,\text{ArcTanh}[c\,x])\,\text{PolyLog}\left[2,-1+\frac{2}{1+c\,x}\right]}{d}-\frac{b^2\,c^2\,\text{PolyLog}\left[3,-1+\frac{2}{1+c\,x}\right]}{2\,d}$$

Result (type 4, 317 leaves):

$$\frac{1}{2\,d}\left(-\frac{a^2}{x^2}+\frac{2\,a^2\,c}{x}+2\,a^2\,c^2\,\text{Log}[x]-2\,a^2\,c^2\,\text{Log}[1+c\,x]+\frac{1}{x^2}2\,a\,b\left(\text{ArcTanh}[c\,x]\left(-1+2\,c\,x+c^2\,x^2+2\,c^2\,x^2\,\text{Log}\left[1-e^{-2\,\text{ArcTanh}[c\,x]}\right]\right)-c\,x\left(1+2\,c\,x\,\text{Log}\left[\frac{c\,x}{\sqrt{1-c^2\,x^2}}\right]\right)-c^2\,x^2\,\text{PolyLog}\left[2,e^{-2\,\text{ArcTanh}[c\,x]}\right]\right)+2\,b^2\,c^2\left(\frac{i\,\pi^3}{24}-\frac{\text{ArcTanh}[c\,x]}{c\,x}-\frac{1}{2}\text{ArcTanh}[c\,x]^2-\frac{\text{ArcTanh}[c\,x]^2}{2\,c^2\,x^2}+\frac{\text{ArcTanh}[c\,x]^2}{c\,x}-\frac{2}{3}\text{ArcTanh}[c\,x]^3-2\,\text{ArcTanh}[c\,x]\,\text{Log}\left[1-e^{-2\,\text{ArcTanh}[c\,x]}\right]+\text{ArcTanh}[c\,x]^2\,\text{Log}\left[1-e^{2\,\text{ArcTanh}[c\,x]}\right]+\text{Log}\left[\frac{c\,x}{\sqrt{1-c^2\,x^2}}\right]+\text{PolyLog}\left[2,e^{-2\,\text{ArcTanh}[c\,x]}\right]+\text{ArcTanh}[c\,x]\,\text{PolyLog}\left[2,e^{2\,\text{ArcTanh}[c\,x]}\right]-\frac{1}{2}\text{PolyLog}\left[3,e^{2\,\text{ArcTanh}[c\,x]}\right]\right)\right)$$

Problem 102: Result unnecessarily involves imaginary or complex numbers.

$$\int\frac{(a+b\,\text{ArcTanh}[c\,x])^2}{x^4\,(d+c\,d\,x)}\,dx$$

Optimal (type 4, 334 leaves, 26 steps):

$$-\frac{b^2 c^2}{3 d x}+\frac{b^2 c^3 \text{ArcTanh}[c x]}{3 d}-\frac{b c (a+b \text{ArcTanh}[c x])}{3 d x^2}+\frac{b c^2 (a+b \text{ArcTanh}[c x])}{d x}+\frac{5 c^3 (a+b \text{ArcTanh}[c x])^2}{6 d}-\frac{(a+b \text{ArcTanh}[c x])^2}{3 d x^3}+\frac{c (a+b \text{ArcTanh}[c x])^2}{2 d x^2}-\frac{c^2 (a+b \text{ArcTanh}[c x])^2}{d x}-\frac{b^2 c^3 \text{Log}[x]}{d}+\frac{b^2 c^3 \text{Log}\left[1-c^2 x^2\right]}{2 d}+\frac{8 b c^3 (a+b \text{ArcTanh}[c x]) \text{Log}\left[2-\frac{2}{1+c x}\right]}{3 d}-\frac{c^3 (a+b \text{ArcTanh}[c x])^2 \text{Log}\left[2-\frac{2}{1+c x}\right]}{d}-\frac{4 b^2 c^3 \text{PolyLog}\left[2,-1+\frac{2}{1+c x}\right]}{3 d}+\frac{b c^3 (a+b \text{ArcTanh}[c x]) \text{PolyLog}\left[2,-1+\frac{2}{1+c x}\right]}{d}+\frac{b^2 c^3 \text{PolyLog}\left[3,-1+\frac{2}{1+c x}\right]}{2 d}$$

Result (type 4, 388 leaves):

$$\frac{1}{24 d}\Bigg(-\frac{8 a^2}{x^3}+\frac{12 a^2 c}{x^2}-\frac{24 a^2 c^2}{x}-24 a^2 c^3 \text{Log}[x]+24 a^2 c^3 \text{Log}[1+c x]-\frac{1}{x^3}8 a b \Big(\text{ArcTanh}[c x] \left(2-3 c x+6 c^2 x^2+3 c^3 x^3+6 c^3 x^3 \text{Log}\left[1-e^{-2 \text{ArcTanh}[c x]}\right]\right)-c x \left(-1+3 c x+c^2 x^2+8 c^2 x^2 \text{Log}\left[\frac{c x}{\sqrt{1-c^2 x^2}}\right]\right)-3 c^3 x^3 \text{PolyLog}\left[2,e^{-2 \text{ArcTanh}[c x]}\right]\Big)+b^2 c^3 \Big(-i \pi^3-\frac{8}{c x}+8 \text{ArcTanh}[c x]-\frac{8 \text{ArcTanh}[c x]}{c^2 x^2}+\frac{24 \text{ArcTanh}[c x]}{c x}+20 \text{ArcTanh}[c x]^2-\frac{8 \text{ArcTanh}[c x]^2}{c^3 x^3}+\frac{12 \text{ArcTanh}[c x]^2}{c^2 x^2}-\frac{24 \text{ArcTanh}[c x]^2}{c x}+16 \text{ArcTanh}[c x]^3+64 \text{ArcTanh}[c x] \text{Log}\left[1-e^{-2 \text{ArcTanh}[c x]}\right]-24 \text{ArcTanh}[c x]^2 \text{Log}\left[1-e^{2 \text{ArcTanh}[c x]}\right]-24 \text{Log}\left[\frac{c x}{\sqrt{1-c^2 x^2}}\right]-32 \text{PolyLog}\left[2,e^{-2 \text{ArcTanh}[c x]}\right]-24 \text{ArcTanh}[c x] \text{PolyLog}\left[2,e^{2 \text{ArcTanh}[c x]}\right]+12 \text{PolyLog}\left[3,e^{2 \text{ArcTanh}[c x]}\right]\Big)\Bigg)$$

Problem 108: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{(a+b \text{ArcTanh}[c x])^2}{x (d+c d x)^2} \, dx$$

Optimal (type 4, 295 leaves, 19 steps):

$$\frac{b^2}{2\,d^2\,(1+c\,x)}-\frac{b^2\,\text{ArcTanh}[c\,x]}{2\,d^2}+\frac{b\,(a+b\,\text{ArcTanh}[c\,x])}{d^2\,(1+c\,x)}-\frac{(a+b\,\text{ArcTanh}[c\,x])^2}{2\,d^2}+$$
$$\frac{(a+b\,\text{ArcTanh}[c\,x])^2}{d^2\,(1+c\,x)}+\frac{2\,(a+b\,\text{ArcTanh}[c\,x])^2\,\text{ArcTanh}\left[1-\frac{2}{1-c\,x}\right]}{d^2}+$$
$$\frac{(a+b\,\text{ArcTanh}[c\,x])^2\,\text{Log}\left[\frac{2}{1+c\,x}\right]}{d^2}-\frac{b\,(a+b\,\text{ArcTanh}[c\,x])\,\text{PolyLog}\left[2,1-\frac{2}{1-c\,x}\right]}{d^2}+$$
$$\frac{b\,(a+b\,\text{ArcTanh}[c\,x])\,\text{PolyLog}\left[2,-1+\frac{2}{1-c\,x}\right]}{d^2}-\frac{b\,(a+b\,\text{ArcTanh}[c\,x])\,\text{PolyLog}\left[2,1-\frac{2}{1+c\,x}\right]}{d^2}+$$
$$\frac{b^2\,\text{PolyLog}\left[3,1-\frac{2}{1-c\,x}\right]}{2\,d^2}-\frac{b^2\,\text{PolyLog}\left[3,-1+\frac{2}{1-c\,x}\right]}{2\,d^2}-\frac{b^2\,\text{PolyLog}\left[3,1-\frac{2}{1+c\,x}\right]}{2\,d^2}$$

Result (type 4, 254 leaves) :

$$\frac{1}{24\,d^2}\left(\frac{24\,a^2}{1+c\,x}+24\,a^2\,\text{Log}[c\,x]-24\,a^2\,\text{Log}[1+c\,x]+\right.$$
$$12\,a\,b\,\left(\text{Cosh}[2\,\text{ArcTanh}[c\,x]]-2\,\text{PolyLog}\left[2,e^{-2\,\text{ArcTanh}[c\,x]}\right]+\right.$$
$$2\,\text{ArcTanh}[c\,x]\,\left(\text{Cosh}[2\,\text{ArcTanh}[c\,x]]+2\,\text{Log}\left[1-e^{-2\,\text{ArcTanh}[c\,x]}\right]-\text{Sinh}[2\,\text{ArcTanh}[c\,x]]\right)-$$
$$\left.\text{Sinh}[2\,\text{ArcTanh}[c\,x]]\right)+b^2\,\left(i\,\pi^3-16\,\text{ArcTanh}[c\,x]^3+6\,\text{Cosh}[2\,\text{ArcTanh}[c\,x]]+\right.$$
$$12\,\text{ArcTanh}[c\,x]\,\text{Cosh}[2\,\text{ArcTanh}[c\,x]]+12\,\text{ArcTanh}[c\,x]^2\,\text{Cosh}[2\,\text{ArcTanh}[c\,x]]+$$
$$24\,\text{ArcTanh}[c\,x]^2\,\text{Log}\left[1-e^{2\,\text{ArcTanh}[c\,x]}\right]+24\,\text{ArcTanh}[c\,x]\,\text{PolyLog}\left[2,e^{2\,\text{ArcTanh}[c\,x]}\right]-$$
$$12\,\text{PolyLog}\left[3,e^{2\,\text{ArcTanh}[c\,x]}\right]-6\,\text{Sinh}[2\,\text{ArcTanh}[c\,x]]-$$
$$\left.\left.12\,\text{ArcTanh}[c\,x]\,\text{Sinh}[2\,\text{ArcTanh}[c\,x]]-12\,\text{ArcTanh}[c\,x]^2\,\text{Sinh}[2\,\text{ArcTanh}[c\,x]]\right)\right)$$

Problem 109: Result unnecessarily involves imaginary or complex numbers.

$$\int\frac{(a+b\,\text{ArcTanh}[c\,x])^2}{x^2\,(d+c\,d\,x)^2}\,dx$$

Optimal (type 4, 371 leaves, 23 steps) :

$$-\frac{b^2 c}{2 d^2 (1+c x)}+\frac{b^2 c \operatorname{ArcTanh}[c x]}{2 d^2}-\frac{b c (a+b \operatorname{ArcTanh}[c x])}{d^2 (1+c x)}+\frac{3 c (a+b \operatorname{ArcTanh}[c x])^2}{2 d^2}-\frac{(a+b \operatorname{ArcTanh}[c x])^2}{d^2 x}-\frac{c (a+b \operatorname{ArcTanh}[c x])^2}{d^2 (1+c x)}-\frac{4 c (a+b \operatorname{ArcTanh}[c x])^2 \operatorname{ArcTanh}\left[1-\frac{2}{1-c x}\right]}{d^2}-\frac{2 c (a+b \operatorname{ArcTanh}[c x])^2 \operatorname{Log}\left[\frac{2}{1+c x}\right]}{d^2}+\frac{2 b c (a+b \operatorname{ArcTanh}[c x]) \operatorname{Log}\left[2-\frac{2}{1+c x}\right]}{d^2}+\frac{2 b c (a+b \operatorname{ArcTanh}[c x]) \operatorname{PolyLog}\left[2,1-\frac{2}{1-c x}\right]}{d^2}-\frac{2 b c (a+b \operatorname{ArcTanh}[c x]) \operatorname{PolyLog}\left[2,-1+\frac{2}{1-c x}\right]}{d^2}+\frac{2 b c (a+b \operatorname{ArcTanh}[c x]) \operatorname{PolyLog}\left[2,1-\frac{2}{1+c x}\right]}{d^2}-\frac{b^2 c \operatorname{PolyLog}\left[2,-1+\frac{2}{1+c x}\right]}{d^2}-\frac{b^2 c \operatorname{PolyLog}\left[3,1-\frac{2}{1-c x}\right]}{d^2}+\frac{b^2 c \operatorname{PolyLog}\left[3,-1+\frac{2}{1-c x}\right]}{d^2}+\frac{b^2 c \operatorname{PolyLog}\left[3,1-\frac{2}{1+c x}\right]}{d^2}$$

Result (type 4, 347 leaves):

$$\frac{1}{12 d^2}\Bigg(-\frac{12 a^2}{x}-\frac{12 a^2 c}{1+c x}-24 a^2 c \operatorname{Log}[x]+24 a^2 c \operatorname{Log}[1+c x]+b^2 c \Bigg(-i \pi^3+12 \operatorname{ArcTanh}[c x]^2-\frac{12 \operatorname{ArcTanh}[c x]^2}{c x}+16 \operatorname{ArcTanh}[c x]^3-3 \operatorname{Cosh}[2 \operatorname{ArcTanh}[c x]]-6 \operatorname{ArcTanh}[c x] \operatorname{Cosh}[2 \operatorname{ArcTanh}[c x]]-6 \operatorname{ArcTanh}[c x]^2 \operatorname{Cosh}[2 \operatorname{ArcTanh}[c x]]+24 \operatorname{ArcTanh}[c x] \operatorname{Log}\left[1-e^{-2 \operatorname{ArcTanh}[c x]}\right]-24 \operatorname{ArcTanh}[c x]^2 \operatorname{Log}\left[1-e^{2 \operatorname{ArcTanh}[c x]}\right]-12 \operatorname{PolyLog}\left[2, e^{-2 \operatorname{ArcTanh}[c x]}\right]-24 \operatorname{ArcTanh}[c x] \operatorname{PolyLog}\left[2, e^{2 \operatorname{ArcTanh}[c x]}\right]+12 \operatorname{PolyLog}\left[3, e^{2 \operatorname{ArcTanh}[c x]}\right]+3 \operatorname{Sinh}[2 \operatorname{ArcTanh}[c x]]+6 \operatorname{ArcTanh}[c x] \operatorname{Sinh}[2 \operatorname{ArcTanh}[c x]]+6 \operatorname{ArcTanh}[c x]^2 \operatorname{Sinh}[2 \operatorname{ArcTanh}[c x]]\Bigg)+6 a b c \Bigg(-\operatorname{Cosh}[2 \operatorname{ArcTanh}[c x]]+4 \operatorname{Log}\left[\frac{c x}{\sqrt{1-c^2 x^2}}\right]+4 \operatorname{PolyLog}\left[2, e^{-2 \operatorname{ArcTanh}[c x]}\right]+\operatorname{Sinh}[2 \operatorname{ArcTanh}[c x]]+\operatorname{ArcTanh}[c x]\left(-\frac{4}{c x}-2 \operatorname{Cosh}[2 \operatorname{ArcTanh}[c x]]-8 \operatorname{Log}\left[1-e^{-2 \operatorname{ArcTanh}[c x]}\right]+2 \operatorname{Sinh}[2 \operatorname{ArcTanh}[c x]]\right)\Bigg)\Bigg)$$

Problem 110: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{(a+b \operatorname{ArcTanh}[c x])^2}{x^3 (d+c d x)^2} \, dx$$

Optimal (type 4, 480 leaves, 31 steps):

$$\frac{b^2 c^2}{2 d^2 (1+c x)}-\frac{b^2 c^2 \text{ArcTanh}[c x]}{2 d^2}-\frac{b c (a+b \text{ArcTanh}[c x])}{d^2 x}+\frac{b c^2 (a+b \text{ArcTanh}[c x])}{d^2 (1+c x)}-\frac{2 c^2 (a+b \text{ArcTanh}[c x])^2}{d^2}-\frac{(a+b \text{ArcTanh}[c x])^2}{2 d^2 x^2}+\frac{2 c (a+b \text{ArcTanh}[c x])^2}{d^2 x}+\frac{c^2 (a+b \text{ArcTanh}[c x])^2}{d^2 (1+c x)}+\frac{6 c^2 (a+b \text{ArcTanh}[c x])^2 \text{ArcTanh}\left[1-\frac{2}{1-c x}\right]}{d^2}+\frac{b^2 c^2 \text{Log}[x]}{d^2}+\frac{3 c^2 (a+b \text{ArcTanh}[c x])^2 \text{Log}\left[\frac{2}{1+c x}\right]}{d^2}-\frac{b^2 c^2 \text{Log}\left[1-c^2 x^2\right]}{2 d^2}-\frac{4 b c^2 (a+b \text{ArcTanh}[c x]) \text{Log}\left[2-\frac{2}{1+c x}\right]}{d^2}-\frac{3 b c^2 (a+b \text{ArcTanh}[c x]) \text{PolyLog}\left[2,1-\frac{2}{1-c x}\right]}{d^2}+\frac{3 b c^2 (a+b \text{ArcTanh}[c x]) \text{PolyLog}\left[2,-1+\frac{2}{1-c x}\right]}{d^2}-\frac{3 b c^2 (a+b \text{ArcTanh}[c x]) \text{PolyLog}\left[2,1-\frac{2}{1+c x}\right]}{d^2}+\frac{2 b^2 c^2 \text{PolyLog}\left[2,-1+\frac{2}{1+c x}\right]}{d^2}+\frac{3 b^2 c^2 \text{PolyLog}\left[3,1-\frac{2}{1-c x}\right]}{2 d^2}-\frac{3 b^2 c^2 \text{PolyLog}\left[3,-1+\frac{2}{1-c x}\right]}{2 d^2}-\frac{3 b^2 c^2 \text{PolyLog}\left[3,1-\frac{2}{1+c x}\right]}{2 d^2}$$

Result (type 4, 452 leaves):

$$\frac{1}{8 d^2}\left(-\frac{4 a^2}{x^2}+\frac{16 a^2 c}{x}+\frac{8 a^2 c^2}{1+c x}+24 a^2 c^2 \text{Log}[x]-24 a^2 c^2 \text{Log}[1+c x]+b^2 c^2 \left(i \pi^3-\frac{8 \text{ArcTanh}[c x]}{c x}-12 \text{ArcTanh}[c x]^2-\frac{4 \text{ArcTanh}[c x]^2}{c^2 x^2}+\frac{16 \text{ArcTanh}[c x]^2}{c x}-16 \text{ArcTanh}[c x]^3+2 \text{Cosh}[2 \text{ArcTanh}[c x]]+4 \text{ArcTanh}[c x] \text{Cosh}[2 \text{ArcTanh}[c x]]+4 \text{ArcTanh}[c x]^2 \text{Cosh}[2 \text{ArcTanh}[c x]]-32 \text{ArcTanh}[c x] \text{Log}\left[1-e^{-2 \text{ArcTanh}[c x]}\right]+24 \text{ArcTanh}[c x]^2 \text{Log}\left[1-e^{2 \text{ArcTanh}[c x]}\right]+8 \text{Log}\left[\frac{c x}{\sqrt{1-c^2 x^2}}\right]+16 \text{PolyLog}\left[2,e^{-2 \text{ArcTanh}[c x]}\right]+24 \text{ArcTanh}[c x] \text{PolyLog}\left[2,e^{2 \text{ArcTanh}[c x]}\right]-12 \text{PolyLog}\left[3,e^{2 \text{ArcTanh}[c x]}\right]-2 \text{Sinh}[2 \text{ArcTanh}[c x]]-4 \text{ArcTanh}[c x] \text{Sinh}[2 \text{ArcTanh}[c x]]-4 \text{ArcTanh}[c x]^2 \text{Sinh}[2 \text{ArcTanh}[c x]]\right)+\frac{1}{x^2}4 a b \left(-6 c^2 x^2 \text{PolyLog}\left[2,e^{-2 \text{ArcTanh}[c x]}\right]+c x \left(-2+c x \text{Cosh}[2 \text{ArcTanh}[c x]]-8 c x \text{Log}\left[\frac{c x}{\sqrt{1-c^2 x^2}}\right]-c x \text{Sinh}[2 \text{ArcTanh}[c x]]\right)+2 \text{ArcTanh}[c x] \left(-1+4 c x+c^2 x^2+c^2 x^2 \text{Cosh}[2 \text{ArcTanh}[c x]]+6 c^2 x^2 \text{Log}\left[1-e^{-2 \text{ArcTanh}[c x]}\right]-c^2 x^2 \text{Sinh}[2 \text{ArcTanh}[c x]]\right)\right)\right)$$

Problem 116: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{(a+b \text{ArcTanh}[c x])^2}{x (d+c d x)^3} \, dx$$

Optimal (type 4, 362 leaves, 32 steps):

$$\frac{b^2}{16 d^3 (1+c x)^2}+\frac{11 b^2}{16 d^3 (1+c x)}-\frac{11 b^2 \text{ArcTanh}[c x]}{16 d^3}+\frac{b (a+b \text{ArcTanh}[c x])}{4 d^3 (1+c x)^2}+\frac{5 b (a+b \text{ArcTanh}[c x])}{4 d^3 (1+c x)}-\frac{5 (a+b \text{ArcTanh}[c x])^2}{8 d^3}+\frac{(a+b \text{ArcTanh}[c x])^2}{2 d^3 (1+c x)^2}+\frac{(a+b \text{ArcTanh}[c x])^2}{d^3 (1+c x)}+\frac{2 (a+b \text{ArcTanh}[c x])^2 \text{ArcTanh}\left[1-\frac{2}{1-c x}\right]}{d^3}+\frac{(a+b \text{ArcTanh}[c x])^2 \text{Log}\left[\frac{2}{1+c x}\right]}{d^3}-\frac{b (a+b \text{ArcTanh}[c x]) \text{PolyLog}\left[2,1-\frac{2}{1-c x}\right]}{d^3}+\frac{b (a+b \text{ArcTanh}[c x]) \text{PolyLog}\left[2,-1+\frac{2}{1-c x}\right]}{d^3}-\frac{b (a+b \text{ArcTanh}[c x]) \text{PolyLog}\left[2,1-\frac{2}{1+c x}\right]}{d^3}+\frac{b^2 \text{PolyLog}\left[3,1-\frac{2}{1-c x}\right]}{2 d^3}-\frac{b^2 \text{PolyLog}\left[3,-1+\frac{2}{1-c x}\right]}{2 d^3}-\frac{b^2 \text{PolyLog}\left[3,1-\frac{2}{1+c x}\right]}{2 d^3}$$

Result (type 4, 376 leaves):

$$\frac{1}{192 d^3}\Bigg(\frac{96 a^2}{(1+c x)^2}+\frac{192 a^2}{1+c x}+192 a^2 \text{Log}[c x]-192 a^2 \text{Log}[1+c x]+12 a b \Big(12 \text{Cosh}[2 \text{ArcTanh}[c x]]+\text{Cosh}[4 \text{ArcTanh}[c x]]-16 \text{PolyLog}\left[2,e^{-2 \text{ArcTanh}[c x]}\right]-12 \text{Sinh}[2 \text{ArcTanh}[c x]]+4 \text{ArcTanh}[c x] \left(6 \text{Cosh}[2 \text{ArcTanh}[c x]]+\text{Cosh}[4 \text{ArcTanh}[c x]]+8 \text{Log}\left[1-e^{-2 \text{ArcTanh}[c x]}\right]-6 \text{Sinh}[2 \text{ArcTanh}[c x]]-\text{Sinh}[4 \text{ArcTanh}[c x]]\right)-\text{Sinh}[4 \text{ArcTanh}[c x]]\Big)+b^2 \Big(8 i \pi^3-128 \text{ArcTanh}[c x]^3+72 \text{Cosh}[2 \text{ArcTanh}[c x]]+144 \text{ArcTanh}[c x] \text{Cosh}[2 \text{ArcTanh}[c x]]+144 \text{ArcTanh}[c x]^2 \text{Cosh}[2 \text{ArcTanh}[c x]]+3 \text{Cosh}[4 \text{ArcTanh}[c x]]+12 \text{ArcTanh}[c x] \text{Cosh}[4 \text{ArcTanh}[c x]]+24 \text{ArcTanh}[c x]^2 \text{Cosh}[4 \text{ArcTanh}[c x]]+192 \text{ArcTanh}[c x]^2 \text{Log}\left[1-e^{2 \text{ArcTanh}[c x]}\right]+192 \text{ArcTanh}[c x] \text{PolyLog}\left[2,e^{2 \text{ArcTanh}[c x]}\right]-96 \text{PolyLog}\left[3,e^{2 \text{ArcTanh}[c x]}\right]-72 \text{Sinh}[2 \text{ArcTanh}[c x]]-144 \text{ArcTanh}[c x] \text{Sinh}[2 \text{ArcTanh}[c x]]-144 \text{ArcTanh}[c x]^2 \text{Sinh}[2 \text{ArcTanh}[c x]]-3 \text{Sinh}[4 \text{ArcTanh}[c x]]-12 \text{ArcTanh}[c x] \text{Sinh}[4 \text{ArcTanh}[c x]]-24 \text{ArcTanh}[c x]^2 \text{Sinh}[4 \text{ArcTanh}[c x]]\Big)\Bigg)$$

Problem 117: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{(a+b \text{ArcTanh}[c x])^2}{x^2 (d+c d x)^3} \, dx$$

Optimal (type 4, 448 leaves, 36 steps):

$$-\frac{b^2 c}{16 d^3 (1+c x)^2}-\frac{19 b^2 c}{16 d^3 (1+c x)}+\frac{19 b^2 c \text{ArcTanh}[c x]}{16 d^3}-\frac{b c (a+b \text{ArcTanh}[c x])}{4 d^3 (1+c x)^2}-\frac{9 b c (a+b \text{ArcTanh}[c x])}{4 d^3 (1+c x)}+\frac{17 c (a+b \text{ArcTanh}[c x])^2}{8 d^3}-\frac{(a+b \text{ArcTanh}[c x])^2}{d^3 x}-\frac{c (a+b \text{ArcTanh}[c x])^2}{2 d^3 (1+c x)^2}-\frac{2 c (a+b \text{ArcTanh}[c x])^2}{d^3 (1+c x)}-\frac{6 c (a+b \text{ArcTanh}[c x])^2 \text{ArcTanh}\left[1-\frac{2}{1-c x}\right]}{d^3}-\frac{3 c (a+b \text{ArcTanh}[c x])^2 \text{Log}\left[\frac{2}{1+c x}\right]}{d^3}+\frac{2 b c (a+b \text{ArcTanh}[c x]) \text{Log}\left[2-\frac{2}{1+c x}\right]}{d^3}+\frac{3 b c (a+b \text{ArcTanh}[c x]) \text{PolyLog}\left[2,1-\frac{2}{1-c x}\right]}{d^3}-\frac{3 b c (a+b \text{ArcTanh}[c x]) \text{PolyLog}\left[2,-1+\frac{2}{1-c x}\right]}{d^3}+\frac{3 b c (a+b \text{ArcTanh}[c x]) \text{PolyLog}\left[2,1-\frac{2}{1+c x}\right]}{d^3}-\frac{b^2 c \text{PolyLog}\left[2,-1+\frac{2}{1+c x}\right]}{d^3}-\frac{3 b^2 c \text{PolyLog}\left[3,1-\frac{2}{1-c x}\right]}{2 d^3}+\frac{3 b^2 c \text{PolyLog}\left[3,-1+\frac{2}{1-c x}\right]}{2 d^3}+\frac{3 b^2 c \text{PolyLog}\left[3,1-\frac{2}{1+c x}\right]}{2 d^3}$$

Result (type 4, 479 leaves) :

$$\frac{1}{64 d^3}\left(-\frac{64 a^2}{x}-\frac{32 a^2 c}{(1+c x)^2}-\frac{128 a^2 c}{1+c x}-192 a^2 c \text{Log}[x]+192 a^2 c \text{Log}[1+c x]+b^2 c \left(-8 i \pi^3+64 \text{ArcTanh}[c x]^2-\frac{64 \text{ArcTanh}[c x]^2}{c x}+128 \text{ArcTanh}[c x]^3-40 \text{Cosh}[2 \text{ArcTanh}[c x]]-80 \text{ArcTanh}[c x] \text{Cosh}[2 \text{ArcTanh}[c x]]-80 \text{ArcTanh}[c x]^2 \text{Cosh}[2 \text{ArcTanh}[c x]]-\text{Cosh}[4 \text{ArcTanh}[c x]]-4 \text{ArcTanh}[c x] \text{Cosh}[4 \text{ArcTanh}[c x]]-8 \text{ArcTanh}[c x]^2 \text{Cosh}[4 \text{ArcTanh}[c x]]+128 \text{ArcTanh}[c x] \text{Log}\left[1-e^{-2 \text{ArcTanh}[c x]}\right]-192 \text{ArcTanh}[c x]^2 \text{Log}\left[1-e^{2 \text{ArcTanh}[c x]}\right]-64 \text{PolyLog}\left[2,e^{-2 \text{ArcTanh}[c x]}\right]-192 \text{ArcTanh}[c x] \text{PolyLog}\left[2,e^{2 \text{ArcTanh}[c x]}\right]+96 \text{PolyLog}\left[3,e^{2 \text{ArcTanh}[c x]}\right]+40 \text{Sinh}[2 \text{ArcTanh}[c x]]+80 \text{ArcTanh}[c x] \text{Sinh}[2 \text{ArcTanh}[c x]]+80 \text{ArcTanh}[c x]^2 \text{Sinh}[2 \text{ArcTanh}[c x]]+\text{Sinh}[4 \text{ArcTanh}[c x]]+4 \text{ArcTanh}[c x] \text{Sinh}[4 \text{ArcTanh}[c x]]+8 \text{ArcTanh}[c x]^2 \text{Sinh}[4 \text{ArcTanh}[c x]]\right)+\frac{1}{x} 4 a b \left(48 c x \text{PolyLog}\left[2,e^{-2 \text{ArcTanh}[c x]}\right]+c x \left(-20 \text{Cosh}[2 \text{ArcTanh}[c x]]-\text{Cosh}[4 \text{ArcTanh}[c x]]+32 \text{Log}\left[\frac{c x}{\sqrt{1-c^2 x^2}}\right]+20 \text{Sinh}[2 \text{ArcTanh}[c x]]+\text{Sinh}[4 \text{ArcTanh}[c x]]\right)-4 \text{ArcTanh}[c x] \left(8+10 c x \text{Cosh}[2 \text{ArcTanh}[c x]]+c x \text{Cosh}[4 \text{ArcTanh}[c x]]+24 c x \text{Log}\left[1-e^{-2 \text{ArcTanh}[c x]}\right]-10 c x \text{Sinh}[2 \text{ArcTanh}[c x]]-c x \text{Sinh}[4 \text{ArcTanh}[c x]]\right)\right)\right)$$

Problem 120: Result more than twice size of optimal antiderivative.

$$\int (1+c x)^3 (a+b \text{ArcTanh}[c x])^3 \, dx$$

Optimal (type 4, 306 leaves, 26 steps):

$$3 a b^2 x+\frac{b^3 x}{4}-\frac{b^3 \text{ArcTanh}[c x]}{4 c}+3 b^3 x \text{ArcTanh}[c x]+\frac{1}{4} b^2 c x^2 (a+b \text{ArcTanh}[c x])+\frac{4 b (a+b \text{ArcTanh}[c x])^2}{c}+\frac{21}{4} b x (a+b \text{ArcTanh}[c x])^2+\frac{3}{2} b c x^2 (a+b \text{ArcTanh}[c x])^2+\frac{1}{4} b c^2 x^3 (a+b \text{ArcTanh}[c x])^2+\frac{(1+c x)^4 (a+b \text{ArcTanh}[c x])^3}{4 c}-\frac{11 b^2 (a+b \text{ArcTanh}[c x]) \text{Log}\left[\frac{2}{1-c x}\right]}{c}-\frac{6 b (a+b \text{ArcTanh}[c x])^2 \text{Log}\left[\frac{2}{1-c x}\right]}{c}+\frac{3 b^3 \text{Log}\left[1-c^2 x^2\right]}{2 c}-\frac{11 b^3 \text{PolyLog}\left[2,1-\frac{2}{1-c x}\right]}{2 c}-\frac{6 b^2 (a+b \text{ArcTanh}[c x]) \text{PolyLog}\left[2,1-\frac{2}{1-c x}\right]}{c}+\frac{3 b^3 \text{PolyLog}\left[3,1-\frac{2}{1-c x}\right]}{c}$$

Result (type 4, 644 leaves):

$$\frac{1}{8 c}\Big(-2 a b^2+8 a^3 c x+42 a^2 b c x+24 a b^2 c x+2 b^3 c x+12 a^3 c^2 x^2+12 a^2 b c^2 x^2+2 a b^2 c^2 x^2+8 a^3 c^3 x^3+2 a^2 b c^3 x^3+2 a^3 c^4 x^4-24 a b^2 \text{ArcTanh}[c x]-2 b^3 \text{ArcTanh}[c x]+24 a^2 b c x \text{ArcTanh}[c x]+84 a b^2 c x \text{ArcTanh}[c x]+24 b^3 c x \text{ArcTanh}[c x]+36 a^2 b c^2 x^2 \text{ArcTanh}[c x]+24 a b^2 c^2 x^2 \text{ArcTanh}[c x]+2 b^3 c^2 x^2 \text{ArcTanh}[c x]+24 a^2 b c^3 x^3 \text{ArcTanh}[c x]+4 a b^2 c^3 x^3 \text{ArcTanh}[c x]+6 a^2 b c^4 x^4 \text{ArcTanh}[c x]-90 a b^2 \text{ArcTanh}[c x]^2-56 b^3 \text{ArcTanh}[c x]^2+24 a b^2 c x \text{ArcTanh}[c x]^2+42 b^3 c x \text{ArcTanh}[c x]^2+36 a b^2 c^2 x^2 \text{ArcTanh}[c x]^2+12 b^3 c^2 x^2 \text{ArcTanh}[c x]^2+24 a b^2 c^3 x^3 \text{ArcTanh}[c x]^2+2 b^3 c^3 x^3 \text{ArcTanh}[c x]^2+6 a b^2 c^4 x^4 \text{ArcTanh}[c x]^2-30 b^3 \text{ArcTanh}[c x]^3+8 b^3 c x \text{ArcTanh}[c x]^3+12 b^3 c^2 x^2 \text{ArcTanh}[c x]^3+8 b^3 c^3 x^3 \text{ArcTanh}[c x]^3+2 b^3 c^4 x^4 \text{ArcTanh}[c x]^3-96 a b^2 \text{ArcTanh}[c x] \text{Log}\left[1+e^{-2 \text{ArcTanh}[c x]}\right]-88 b^3 \text{ArcTanh}[c x] \text{Log}\left[1+e^{-2 \text{ArcTanh}[c x]}\right]-48 b^3 \text{ArcTanh}[c x]^2 \text{Log}\left[1+e^{-2 \text{ArcTanh}[c x]}\right]+45 a^2 b \text{Log}[1-c x]+3 a^2 b \text{Log}[1+c x]+44 a b^2 \text{Log}\left[1-c^2 x^2\right]+12 b^3 \text{Log}\left[1-c^2 x^2\right]+4 b^2 (12 a+11 b+12 b \text{ArcTanh}[c x]) \text{PolyLog}\left[2,-e^{-2 \text{ArcTanh}[c x]}\right]+24 b^3 \text{PolyLog}\left[3,-e^{-2 \text{ArcTanh}[c x]}\right]\Big)$$

Problem 121: Result more than twice size of optimal antiderivative.

$$\int (1+c x)^2 (a+b \text{ArcTanh}[c x])^3 \, dx$$

Optimal (type 4, 240 leaves, 17 steps):

$$a\,b^2\,x+b^3\,x\,\text{ArcTanh}[c\,x]+\frac{5\,b\,(a+b\,\text{ArcTanh}[c\,x])^2}{2\,c}+3\,b\,x\,(a+b\,\text{ArcTanh}[c\,x])^2+\frac{1}{2}\,b\,c\,x^2\,(a+b\,\text{ArcTanh}[c\,x])^2+\frac{(1+c\,x)^3\,(a+b\,\text{ArcTanh}[c\,x])^3}{3\,c}-\frac{6\,b^2\,(a+b\,\text{ArcTanh}[c\,x])\,\text{Log}\left[\frac{2}{1-c\,x}\right]}{c}-\frac{4\,b\,(a+b\,\text{ArcTanh}[c\,x])^2\,\text{Log}\left[\frac{2}{1-c\,x}\right]}{c}+\frac{b^3\,\text{Log}\left[1-c^2\,x^2\right]}{2\,c}-\frac{3\,b^3\,\text{PolyLog}\left[2,1-\frac{2}{1-c\,x}\right]}{c}-\frac{4\,b^2\,(a+b\,\text{ArcTanh}[c\,x])\,\text{PolyLog}\left[2,1-\frac{2}{1-c\,x}\right]}{c}+\frac{2\,b^3\,\text{PolyLog}\left[3,1-\frac{2}{1-c\,x}\right]}{c}$$

Result (type 4, 488 leaves) :

$$\frac{1}{6\,c}\Big(6\,a^3\,c\,x+18\,a^2\,b\,c\,x+6\,a\,b^2\,c\,x+6\,a^3\,c^2\,x^2+3\,a^2\,b\,c^2\,x^2+2\,a^3\,c^3\,x^3-6\,a\,b^2\,\text{ArcTanh}[c\,x]+18\,a^2\,b\,c\,x\,\text{ArcTanh}[c\,x]+36\,a\,b^2\,c\,x\,\text{ArcTanh}[c\,x]+6\,b^3\,c\,x\,\text{ArcTanh}[c\,x]+18\,a^2\,b\,c^2\,x^2\,\text{ArcTanh}[c\,x]+6\,a\,b^2\,c^2\,x^2\,\text{ArcTanh}[c\,x]+6\,a^2\,b\,c^3\,x^3\,\text{ArcTanh}[c\,x]-42\,a\,b^2\,\text{ArcTanh}[c\,x]^2-21\,b^3\,\text{ArcTanh}[c\,x]^2+18\,a\,b^2\,c\,x\,\text{ArcTanh}[c\,x]^2+18\,b^3\,c\,x\,\text{ArcTanh}[c\,x]^2+18\,a\,b^2\,c^2\,x^2\,\text{ArcTanh}[c\,x]^2+3\,b^3\,c^2\,x^2\,\text{ArcTanh}[c\,x]^2+6\,a\,b^2\,c^3\,x^3\,\text{ArcTanh}[c\,x]^2-14\,b^3\,\text{ArcTanh}[c\,x]^3+6\,b^3\,c\,x\,\text{ArcTanh}[c\,x]^3+6\,b^3\,c^2\,x^2\,\text{ArcTanh}[c\,x]^3+2\,b^3\,c^3\,x^3\,\text{ArcTanh}[c\,x]^3-48\,a\,b^2\,\text{ArcTanh}[c\,x]\,\text{Log}\left[1+e^{-2\,\text{ArcTanh}[c\,x]}\right]-36\,b^3\,\text{ArcTanh}[c\,x]\,\text{Log}\left[1+e^{-2\,\text{ArcTanh}[c\,x]}\right]-24\,b^3\,\text{ArcTanh}[c\,x]^2\,\text{Log}\left[1+e^{-2\,\text{ArcTanh}[c\,x]}\right]+21\,a^2\,b\,\text{Log}[1-c\,x]+3\,a^2\,b\,\text{Log}[1+c\,x]+18\,a\,b^2\,\text{Log}\left[1-c^2\,x^2\right]+3\,b^3\,\text{Log}\left[1-c^2\,x^2\right]+6\,b^2\,(4\,a+3\,b+4\,b\,\text{ArcTanh}[c\,x])\,\text{PolyLog}\left[2,-e^{-2\,\text{ArcTanh}[c\,x]}\right]+12\,b^3\,\text{PolyLog}\left[3,-e^{-2\,\text{ArcTanh}[c\,x]}\right]\Big)$$

Problem 132: Result unnecessarily involves imaginary or complex numbers.

$$\int\frac{\text{ArcTanh}[a\,x]^3}{x^2\,(c+a\,c\,x)}\,dx$$

Optimal (type 4, 191 leaves, 10 steps) :

$$\frac{a\,\text{ArcTanh}[a\,x]^3}{c}-\frac{\text{ArcTanh}[a\,x]^3}{c\,x}+\frac{3\,a\,\text{ArcTanh}[a\,x]^2\,\text{Log}\left[2-\frac{2}{1+a\,x}\right]}{c}-\frac{a\,\text{ArcTanh}[a\,x]^3\,\text{Log}\left[2-\frac{2}{1+a\,x}\right]}{c}-\frac{3\,a\,\text{ArcTanh}[a\,x]\,\text{PolyLog}\left[2,-1+\frac{2}{1+a\,x}\right]}{c}+\frac{3\,a\,\text{ArcTanh}[a\,x]^2\,\text{PolyLog}\left[2,-1+\frac{2}{1+a\,x}\right]}{2\,c}-\frac{3\,a\,\text{PolyLog}\left[3,-1+\frac{2}{1+a\,x}\right]}{2\,c}+\frac{3\,a\,\text{ArcTanh}[a\,x]\,\text{PolyLog}\left[3,-1+\frac{2}{1+a\,x}\right]}{2\,c}+\frac{3\,a\,\text{PolyLog}\left[4,-1+\frac{2}{1+a\,x}\right]}{4\,c}$$

Result (type 4, 154 leaves) :

$$\frac{1}{c}a\left(\frac{i\pi^3}{8}-\frac{\pi^4}{64}-\text{ArcTanh}[a x]^3-\frac{\text{ArcTanh}[a x]^3}{a x}+\frac{1}{2}\text{ArcTanh}[a x]^4+3\,\text{ArcTanh}[a x]^2\,\text{Log}\left[1-e^{2\,\text{ArcTanh}[a x]}\right]-\text{ArcTanh}[a x]^3\,\text{Log}\left[1-e^{2\,\text{ArcTanh}[a x]}\right]-\frac{3}{2}\left(-2+\text{ArcTanh}[a x]\right)\text{ArcTanh}[a x]\,\text{PolyLog}\left[2,e^{2\,\text{ArcTanh}[a x]}\right]+\frac{3}{2}\left(-1+\text{ArcTanh}[a x]\right)\text{PolyLog}\left[3,e^{2\,\text{ArcTanh}[a x]}\right]-\frac{3}{4}\text{PolyLog}\left[4,e^{2\,\text{ArcTanh}[a x]}\right]\right)$$

Problem 133: Result unnecessarily involves imaginary or complex numbers.

$$\int\frac{\text{ArcTanh}[a x]^3}{x^3\,(c+a c x)}\,dx$$

Optimal (type 4, 305 leaves, 18 steps):

$$\frac{3a^2\,\text{ArcTanh}[a x]^2}{2c}-\frac{3a\,\text{ArcTanh}[a x]^2}{2cx}-\frac{a^2\,\text{ArcTanh}[a x]^3}{2c}-\frac{\text{ArcTanh}[a x]^3}{2cx^2}+\frac{a\,\text{ArcTanh}[a x]^3}{cx}+\frac{3a^2\,\text{ArcTanh}[a x]\,\text{Log}\left[2-\frac{2}{1+a x}\right]}{c}-\frac{3a^2\,\text{ArcTanh}[a x]^2\,\text{Log}\left[2-\frac{2}{1+a x}\right]}{c}+\frac{a^2\,\text{ArcTanh}[a x]^3\,\text{Log}\left[2-\frac{2}{1+a x}\right]}{c}-\frac{3a^2\,\text{PolyLog}\left[2,-1+\frac{2}{1+a x}\right]}{2c}+\frac{3a^2\,\text{ArcTanh}[a x]\,\text{PolyLog}\left[2,-1+\frac{2}{1+a x}\right]}{c}-\frac{3a^2\,\text{ArcTanh}[a x]^2\,\text{PolyLog}\left[2,-1+\frac{2}{1+a x}\right]}{2c}+\frac{3a^2\,\text{PolyLog}\left[3,-1+\frac{2}{1+a x}\right]}{2c}-\frac{3a^2\,\text{ArcTanh}[a x]\,\text{PolyLog}\left[3,-1+\frac{2}{1+a x}\right]}{2c}-\frac{3a^2\,\text{PolyLog}\left[4,-1+\frac{2}{1+a x}\right]}{4c}$$

Result (type 4, 222 leaves):

$$\frac{1}{64c}a^2\left(-8i\pi^3+\pi^4+96\,\text{ArcTanh}[a x]^2-\frac{96\,\text{ArcTanh}[a x]^2}{a x}+96\,\text{ArcTanh}[a x]^3-\frac{32\,\text{ArcTanh}[a x]^3}{a^2x^2}+\frac{64\,\text{ArcTanh}[a x]^3}{a x}-32\,\text{ArcTanh}[a x]^4+192\,\text{ArcTanh}[a x]\,\text{Log}\left[1-e^{-2\,\text{ArcTanh}[a x]}\right]-192\,\text{ArcTanh}[a x]^2\,\text{Log}\left[1-e^{2\,\text{ArcTanh}[a x]}\right]+64\,\text{ArcTanh}[a x]^3\,\text{Log}\left[1-e^{2\,\text{ArcTanh}[a x]}\right]-96\,\text{PolyLog}\left[2,e^{-2\,\text{ArcTanh}[a x]}\right]+96\left(-2+\text{ArcTanh}[a x]\right)\text{ArcTanh}[a x]\,\text{PolyLog}\left[2,e^{2\,\text{ArcTanh}[a x]}\right]+96\,\text{PolyLog}\left[3,e^{2\,\text{ArcTanh}[a x]}\right]-96\,\text{ArcTanh}[a x]\,\text{PolyLog}\left[3,e^{2\,\text{ArcTanh}[a x]}\right]+48\,\text{PolyLog}\left[4,e^{2\,\text{ArcTanh}[a x]}\right]\right)$$

Problem 139: Result unnecessarily involves imaginary or complex numbers.

$$\int\frac{\text{ArcTanh}[a x]^4}{x^2\,(c-a c x)}\,dx$$

Optimal (type 4, 239 leaves, 12 steps):

$$\frac{a\,\text{ArcTanh}[a x]^4}{c}-\frac{\text{ArcTanh}[a x]^4}{c x}+\frac{a\,\text{ArcTanh}[a x]^4\,\text{Log}\left[2-\frac{2}{1-a x}\right]}{c}+\frac{4\,a\,\text{ArcTanh}[a x]^3\,\text{Log}\left[2-\frac{2}{1+a x}\right]}{c}+\frac{2\,a\,\text{ArcTanh}[a x]^3\,\text{PolyLog}\left[2,-1+\frac{2}{1-a x}\right]}{c}-\frac{6\,a\,\text{ArcTanh}[a x]^2\,\text{PolyLog}\left[2,-1+\frac{2}{1+a x}\right]}{c}-\frac{3\,a\,\text{ArcTanh}[a x]^2\,\text{PolyLog}\left[3,-1+\frac{2}{1-a x}\right]}{c}-\frac{6\,a\,\text{ArcTanh}[a x]\,\text{PolyLog}\left[3,-1+\frac{2}{1+a x}\right]}{c}+\frac{3\,a\,\text{ArcTanh}[a x]\,\text{PolyLog}\left[4,-1+\frac{2}{1-a x}\right]}{c}-\frac{3\,a\,\text{PolyLog}\left[4,-1+\frac{2}{1+a x}\right]}{c}-\frac{3\,a\,\text{PolyLog}\left[5,-1+\frac{2}{1-a x}\right]}{2 c}$$

Result (type 4, 172 leaves):

$$-\frac{1}{c}a\left(-\frac{\pi^4}{16}+\frac{i\pi^5}{160}+\text{ArcTanh}[a x]^4+\frac{\text{ArcTanh}[a x]^4}{a x}-4\,\text{ArcTanh}[a x]^3\,\text{Log}\left[1-e^{2\,\text{ArcTanh}[a x]}\right]-\text{ArcTanh}[a x]^4\,\text{Log}\left[1-e^{2\,\text{ArcTanh}[a x]}\right]-2\,\text{ArcTanh}[a x]^2\,(3+\text{ArcTanh}[a x])\,\text{PolyLog}\left[2,e^{2\,\text{ArcTanh}[a x]}\right]+3\,\text{ArcTanh}[a x]\,(2+\text{ArcTanh}[a x])\,\text{PolyLog}\left[3,e^{2\,\text{ArcTanh}[a x]}\right]-3\,\text{PolyLog}\left[4,e^{2\,\text{ArcTanh}[a x]}\right]-3\,\text{ArcTanh}[a x]\,\text{PolyLog}\left[4,e^{2\,\text{ArcTanh}[a x]}\right]+\frac{3}{2}\text{PolyLog}\left[5,e^{2\,\text{ArcTanh}[a x]}\right]\right)$$

Problem 140: Result unnecessarily involves imaginary or complex numbers.

$$\int\frac{\text{ArcTanh}[a x]^4}{x^3\,(c-a c x)}\,dx$$

Optimal (type 4, 380 leaves, 21 steps):

$$\frac{2 a^2\,\text{ArcTanh}[a x]^3}{c}-\frac{2\,a\,\text{ArcTanh}[a x]^3}{c x}+\frac{3 a^2\,\text{ArcTanh}[a x]^4}{2 c}-\frac{\text{ArcTanh}[a x]^4}{2 c x^2}-\frac{a\,\text{ArcTanh}[a x]^4}{c x}+\frac{a^2\,\text{ArcTanh}[a x]^4\,\text{Log}\left[2-\frac{2}{1-a x}\right]}{c}+\frac{6 a^2\,\text{ArcTanh}[a x]^2\,\text{Log}\left[2-\frac{2}{1+a x}\right]}{c}+\frac{4 a^2\,\text{ArcTanh}[a x]^3\,\text{Log}\left[2-\frac{2}{1+a x}\right]}{c}+\frac{2 a^2\,\text{ArcTanh}[a x]^3\,\text{PolyLog}\left[2,-1+\frac{2}{1-a x}\right]}{c}-\frac{6 a^2\,\text{ArcTanh}[a x]\,\text{PolyLog}\left[2,-1+\frac{2}{1+a x}\right]}{c}-\frac{6 a^2\,\text{ArcTanh}[a x]^2\,\text{PolyLog}\left[2,-1+\frac{2}{1+a x}\right]}{c}-\frac{3 a^2\,\text{ArcTanh}[a x]^2\,\text{PolyLog}\left[3,-1+\frac{2}{1-a x}\right]}{c}-\frac{3 a^2\,\text{PolyLog}\left[3,-1+\frac{2}{1+a x}\right]}{c}-\frac{6 a^2\,\text{ArcTanh}[a x]\,\text{PolyLog}\left[3,-1+\frac{2}{1+a x}\right]}{c}+\frac{3 a^2\,\text{ArcTanh}[a x]\,\text{PolyLog}\left[4,-1+\frac{2}{1-a x}\right]}{c}-\frac{3 a^2\,\text{PolyLog}\left[4,-1+\frac{2}{1+a x}\right]}{c}-\frac{3 a^2\,\text{PolyLog}\left[5,-1+\frac{2}{1-a x}\right]}{2 c}$$

Result (type 4, 250 leaves):

$$-\frac{1}{c}a^2\left(-\frac{i\pi^3}{4}-\frac{\pi^4}{16}+\frac{i\pi^5}{160}+2\,\text{ArcTanh}[a\,x]^3+\frac{2\,\text{ArcTanh}[a\,x]^3}{a\,x}+\frac{1}{2}\text{ArcTanh}[a\,x]^4+\frac{\text{ArcTanh}[a\,x]^4}{2\,a^2\,x^2}+\frac{\text{ArcTanh}[a\,x]^4}{a\,x}-6\,\text{ArcTanh}[a\,x]^2\,\text{Log}\left[1-e^{2\,\text{ArcTanh}[a\,x]}\right]-4\,\text{ArcTanh}[a\,x]^3\,\text{Log}\left[1-e^{2\,\text{ArcTanh}[a\,x]}\right]-\text{ArcTanh}[a\,x]^4\,\text{Log}\left[1-e^{2\,\text{ArcTanh}[a\,x]}\right]-2\,\text{ArcTanh}[a\,x]\left(3+3\,\text{ArcTanh}[a\,x]+\text{ArcTanh}[a\,x]^2\right)\text{PolyLog}\left[2,e^{2\,\text{ArcTanh}[a\,x]}\right]+3\left(1+\text{ArcTanh}[a\,x]\right)^2\text{PolyLog}\left[3,e^{2\,\text{ArcTanh}[a\,x]}\right]-3\,\text{PolyLog}\left[4,e^{2\,\text{ArcTanh}[a\,x]}\right]-3\,\text{ArcTanh}[a\,x]\,\text{PolyLog}\left[4,e^{2\,\text{ArcTanh}[a\,x]}\right]+\frac{3}{2}\text{PolyLog}\left[5,e^{2\,\text{ArcTanh}[a\,x]}\right]\right)$$

Problem 147: Result unnecessarily involves imaginary or complex numbers.

$$\int\frac{x^3\left(a+b\,\text{ArcTanh}[c\,x]\right)}{d+e\,x}\,dx$$

Optimal (type 4, 275 leaves, 16 steps):

$$\frac{a\,d^2\,x}{e^3}-\frac{b\,d\,x}{2\,c\,e^2}+\frac{b\,x^2}{6\,c\,e}+\frac{b\,d\,\text{ArcTanh}[c\,x]}{2\,c^2\,e^2}+\frac{b\,d^2\,x\,\text{ArcTanh}[c\,x]}{e^3}-\frac{d\,x^2\left(a+b\,\text{ArcTanh}[c\,x]\right)}{2\,e^2}+\frac{x^3\left(a+b\,\text{ArcTanh}[c\,x]\right)}{3\,e}+\frac{d^3\left(a+b\,\text{ArcTanh}[c\,x]\right)\text{Log}\left[\frac{2}{1+c\,x}\right]}{e^4}-\frac{d^3\left(a+b\,\text{ArcTanh}[c\,x]\right)\text{Log}\left[\frac{2\,c\,(d+e\,x)}{(c\,d+e)\,(1+c\,x)}\right]}{e^4}+\frac{b\,d^2\,\text{Log}\left[1-c^2\,x^2\right]}{2\,c\,e^3}+\frac{b\,\text{Log}\left[1-c^2\,x^2\right]}{6\,c^3\,e}-\frac{b\,d^3\,\text{PolyLog}\left[2,1-\frac{2}{1+c\,x}\right]}{2\,e^4}+\frac{b\,d^3\,\text{PolyLog}\left[2,1-\frac{2\,c\,(d+e\,x)}{(c\,d+e)\,(1+c\,x)}\right]}{2\,e^4}$$

Result (type 4, 474 leaves):

$$\frac{1}{12 e^4}\left(-\frac{2 b e^3}{c^3}+12 a d^2 e x-\frac{6 b d e^2 x}{c}-6 a d e^2 x^2+\frac{2 b e^3 x^2}{c}+4 a e^3 x^3+\frac{6 b d e^2 \text{ArcTanh}[c x]}{c^2}-6 i b d^3 \pi \text{ArcTanh}[c x]+12 b d^2 e x \text{ArcTanh}[c x]-6 b d e^2 x^2 \text{ArcTanh}[c x]+4 b e^3 x^3 \text{ArcTanh}[c x]-12 b d^3 \text{ArcTanh}\left[\frac{c d}{e}\right] \text{ArcTanh}[c x]+6 b d^3 \text{ArcTanh}[c x]^2-\frac{6 b d^2 e \text{ArcTanh}[c x]^2}{c}+\frac{6 b d^2 \sqrt{1-\frac{c^2 d^2}{e^2}}\, e\, e^{-\text{ArcTanh}\left[\frac{c d}{e}\right]} \text{ArcTanh}[c x]^2}{c}+12 b d^3 \text{ArcTanh}[c x] \text{Log}\left[1+e^{-2 \text{ArcTanh}[c x]}\right]+6 i b d^3 \pi \text{Log}\left[1+e^{2 \text{ArcTanh}[c x]}\right]-12 b d^3 \text{ArcTanh}\left[\frac{c d}{e}\right] \text{Log}\left[1-e^{-2 \left(\text{ArcTanh}\left[\frac{c d}{e}\right]+\text{ArcTanh}[c x]\right)}\right]-12 b d^3 \text{ArcTanh}[c x] \text{Log}\left[1-e^{-2 \left(\text{ArcTanh}\left[\frac{c d}{e}\right]+\text{ArcTanh}[c x]\right)}\right]-12 a d^3 \text{Log}[d+e x]+\frac{6 b d^2 e \text{Log}\left[1-c^2 x^2\right]}{c}+\frac{2 b e^3 \text{Log}\left[1-c^2 x^2\right]}{c^3}+3 i b d^3 \pi \text{Log}\left[1-c^2 x^2\right]+12 b d^3 \text{ArcTanh}\left[\frac{c d}{e}\right] \text{Log}\left[i \text{Sinh}\left[\text{ArcTanh}\left[\frac{c d}{e}\right]+\text{ArcTanh}[c x]\right]\right]-6 b d^3 \text{PolyLog}\left[2,-e^{-2 \text{ArcTanh}[c x]}\right]+6 b d^3 \text{PolyLog}\left[2, e^{-2 \left(\text{ArcTanh}\left[\frac{c d}{e}\right]+\text{ArcTanh}[c x]\right)}\right]\right)$$

Problem 148: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{x^2 (a+b \text{ArcTanh}[c x])}{d+e x} \, dx$$

Optimal (type 4, 214 leaves, 12 steps):

$$-\frac{a d x}{e^2}+\frac{b x}{2 c e}-\frac{b \text{ArcTanh}[c x]}{2 c^2 e}-\frac{b d x \text{ArcTanh}[c x]}{e^2}+\frac{x^2 (a+b \text{ArcTanh}[c x])}{2 e}-\frac{d^2 (a+b \text{ArcTanh}[c x]) \text{Log}\left[\frac{2}{1+c x}\right]}{e^3}+\frac{d^2 (a+b \text{ArcTanh}[c x]) \text{Log}\left[\frac{2 c (d+e x)}{(c d+e) (1+c x)}\right]}{e^3}-\frac{b d \text{Log}\left[1-c^2 x^2\right]}{2 c e^2}+\frac{b d^2 \text{PolyLog}\left[2,1-\frac{2}{1+c x}\right]}{2 e^3}-\frac{b d^2 \text{PolyLog}\left[2,1-\frac{2 c (d+e x)}{(c d+e) (1+c x)}\right]}{2 e^3}$$

Result (type 4, 394 leaves):

$$\frac{1}{2e^3}\left(-2 a d e x+\frac{b e^2 x}{c}+a e^2 x^2-\frac{b e^2 \text{ArcTanh}[c x]}{c^2}+\right.$$
$$i b d^2 \pi \text{ArcTanh}[c x]-2 b d e x \text{ArcTanh}[c x]+b e^2 x^2 \text{ArcTanh}[c x]+$$
$$2 b d^2 \text{ArcTanh}\left[\frac{c d}{e}\right] \text{ArcTanh}[c x]-b d^2 \text{ArcTanh}[c x]^2+\frac{b d e \text{ArcTanh}[c x]^2}{c}-$$
$$\frac{b d \sqrt{1-\frac{c^2 d^2}{e^2}}\ e\ e^{-\text{ArcTanh}\left[\frac{c d}{e}\right]} \text{ArcTanh}[c x]^2}{c}-2 b d^2 \text{ArcTanh}[c x] \text{Log}\left[1+e^{-2 \text{ArcTanh}[c x]}\right]-$$
$$i b d^2 \pi \text{Log}\left[1+e^{2 \text{ArcTanh}[c x]}\right]+2 b d^2 \text{ArcTanh}\left[\frac{c d}{e}\right] \text{Log}\left[1-e^{-2\left(\text{ArcTanh}\left[\frac{c d}{e}\right]+\text{ArcTanh}[c x]\right)}\right]+$$
$$2 b d^2 \text{ArcTanh}[c x] \text{Log}\left[1-e^{-2\left(\text{ArcTanh}\left[\frac{c d}{e}\right]+\text{ArcTanh}[c x]\right)}\right]+2 a d^2 \text{Log}[d+e x]-\frac{b d e \text{Log}\left[1-c^2 x^2\right]}{c}-$$
$$\frac{1}{2} i b d^2 \pi \text{Log}\left[1-c^2 x^2\right]-2 b d^2 \text{ArcTanh}\left[\frac{c d}{e}\right] \text{Log}\left[i \text{Sinh}\left[\text{ArcTanh}\left[\frac{c d}{e}\right]+\text{ArcTanh}[c x]\right]\right]+$$
$$\left.b d^2 \text{PolyLog}\left[2,-e^{-2 \text{ArcTanh}[c x]}\right]-b d^2 \text{PolyLog}\left[2, e^{-2\left(\text{ArcTanh}\left[\frac{c d}{e}\right]+\text{ArcTanh}[c x]\right)}\right]\right)$$

Problem 149: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.

$$\int \frac{x\,(a+b \text{ArcTanh}[c x])}{d+e x}\,dx$$

Optimal (type 4, 156 leaves, 9 steps):

$$\frac{a x}{e}+\frac{b x \text{ArcTanh}[c x]}{e}+\frac{d\,(a+b \text{ArcTanh}[c x]) \text{Log}\left[\frac{2}{1+c x}\right]}{e^2}-$$
$$\frac{d\,(a+b \text{ArcTanh}[c x]) \text{Log}\left[\frac{2 c (d+e x)}{(c d+e)(1+c x)}\right]}{e^2}+\frac{b \text{Log}\left[1-c^2 x^2\right]}{2 c e}-$$
$$\frac{b d \text{PolyLog}\left[2,1-\frac{2}{1+c x}\right]}{2 e^2}+\frac{b d \text{PolyLog}\left[2,1-\frac{2 c (d+e x)}{(c d+e)(1+c x)}\right]}{2 e^2}$$

Result (type 4, 315 leaves):

$$\frac{1}{2 e^2}\left(2 a e x - 2 a d \text{Log}[d + e x] + \frac{1}{c} b \left(-i c d \pi \text{ArcTanh}[c x] + 2 c e x \text{ArcTanh}[c x] - 2 c d \text{ArcTanh}\left[\frac{c d}{e}\right] \text{ArcTanh}[c x] + c d \text{ArcTanh}[c x]^2 - e \text{ArcTanh}[c x]^2 + \sqrt{1 - \frac{c^2 d^2}{e^2}}\, e\, e^{-\text{ArcTanh}\left[\frac{c d}{e}\right]} \text{ArcTanh}[c x]^2 + 2 c d \text{ArcTanh}[c x] \text{Log}\left[1 + e^{-2 \text{ArcTanh}[c x]}\right] + i c d \pi \text{Log}\left[1 + e^{2 \text{ArcTanh}[c x]}\right] - 2 c d \text{ArcTanh}\left[\frac{c d}{e}\right] \text{Log}\left[1 - e^{-2 \left(\text{ArcTanh}\left[\frac{c d}{e}\right] + \text{ArcTanh}[c x]\right)}\right] - 2 c d \text{ArcTanh}[c x] \text{Log}\left[1 - e^{-2 \left(\text{ArcTanh}\left[\frac{c d}{e}\right] + \text{ArcTanh}[c x]\right)}\right] + e \text{Log}\left[1 - c^2 x^2\right] + \frac{1}{2} i c d \pi \text{Log}\left[1 - c^2 x^2\right] + 2 c d \text{ArcTanh}\left[\frac{c d}{e}\right] \text{Log}\left[i \text{Sinh}\left[\text{ArcTanh}\left[\frac{c d}{e}\right] + \text{ArcTanh}[c x]\right]\right] - c d \text{PolyLog}\left[2, -e^{-2 \text{ArcTanh}[c x]}\right] + c d \text{PolyLog}\left[2, e^{-2 \left(\text{ArcTanh}\left[\frac{c d}{e}\right] + \text{ArcTanh}[c x]\right)}\right]\right)\right)$$

Problem 150: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.

$$\int \frac{a + b \text{ArcTanh}[c x]}{d + e x} \, dx$$

Optimal (type 4, 114 leaves, 4 steps):

$$-\frac{(a + b \text{ArcTanh}[c x]) \text{Log}\left[\frac{2}{1+c x}\right]}{e} + \frac{(a + b \text{ArcTanh}[c x]) \text{Log}\left[\frac{2 c (d+e x)}{(c d+e) (1+c x)}\right]}{e} + \frac{b \text{PolyLog}\left[2, 1 - \frac{2}{1+c x}\right]}{2 e} - \frac{b \text{PolyLog}\left[2, 1 - \frac{2 c (d+e x)}{(c d+e) (1+c x)}\right]}{2 e}$$

Result (type 4, 257 leaves):

$$\frac{1}{e}\left(a\,\text{Log}[d+e\,x]+b\,\text{ArcTanh}[c\,x]\left(\frac{1}{2}\text{Log}\left[1-c^2x^2\right]+\text{Log}\left[i\,\text{Sinh}\left[\text{ArcTanh}\left[\frac{c\,d}{e}\right]+\text{ArcTanh}[c\,x]\right]\right]\right)-\right.$$
$$\frac{1}{2}i\,b\left(-\frac{1}{4}i\,(\pi-2\,i\,\text{ArcTanh}[c\,x])^2+i\left(\text{ArcTanh}\left[\frac{c\,d}{e}\right]+\text{ArcTanh}[c\,x]\right)^2+\right.$$
$$(\pi-2\,i\,\text{ArcTanh}[c\,x])\,\text{Log}\left[1+e^{2\,\text{ArcTanh}[c\,x]}\right]+2\,i\left(\text{ArcTanh}\left[\frac{c\,d}{e}\right]+\text{ArcTanh}[c\,x]\right)$$
$$\text{Log}\left[1-e^{-2\left(\text{ArcTanh}\left[\frac{c\,d}{e}\right]+\text{ArcTanh}[c\,x]\right)}\right]-(\pi-2\,i\,\text{ArcTanh}[c\,x])\,\text{Log}\left[\frac{2}{\sqrt{1-c^2x^2}}\right]-$$
$$2\,i\left(\text{ArcTanh}\left[\frac{c\,d}{e}\right]+\text{ArcTanh}[c\,x]\right)\text{Log}\left[2\,i\,\text{Sinh}\left[\text{ArcTanh}\left[\frac{c\,d}{e}\right]+\text{ArcTanh}[c\,x]\right]\right]-$$
$$\left.\left.i\,\text{PolyLog}\left[2,-e^{2\,\text{ArcTanh}[c\,x]}\right]-i\,\text{PolyLog}\left[2,e^{-2\left(\text{ArcTanh}\left[\frac{c\,d}{e}\right]+\text{ArcTanh}[c\,x]\right)}\right]\right)\right)$$

Problem 151: Result unnecessarily involves imaginary or complex numbers.

$$\int\frac{a+b\,\text{ArcTanh}[c\,x]}{x\,(d+e\,x)}\,dx$$

Optimal (type 4, 148 leaves, 7 steps):

$$\frac{a\,\text{Log}[x]}{d}+\frac{(a+b\,\text{ArcTanh}[c\,x])\,\text{Log}\left[\frac{2}{1+c\,x}\right]}{d}-\frac{(a+b\,\text{ArcTanh}[c\,x])\,\text{Log}\left[\frac{2\,c\,(d+e\,x)}{(c\,d+e)\,(1+c\,x)}\right]}{d}-$$
$$\frac{b\,\text{PolyLog}[2,-c\,x]}{2\,d}+\frac{b\,\text{PolyLog}[2,c\,x]}{2\,d}-\frac{b\,\text{PolyLog}\left[2,1-\frac{2}{1+c\,x}\right]}{2\,d}+\frac{b\,\text{PolyLog}\left[2,1-\frac{2\,c\,(d+e\,x)}{(c\,d+e)\,(1+c\,x)}\right]}{2\,d}$$

Result (type 4, 294 leaves):

$$\frac{1}{2\,d^2}\left(2\,a\,d\,\text{Log}[x]-2\,a\,d\,\text{Log}[d+e\,x]+\right.$$
$$\frac{1}{c}b\left(-i\,c\,d\,\pi\,\text{ArcTanh}[c\,x]-2\,c\,d\,\text{ArcTanh}\left[\frac{c\,d}{e}\right]\text{ArcTanh}[c\,x]+c\,d\,\text{ArcTanh}[c\,x]^2-\right.$$
$$e\,\text{ArcTanh}[c\,x]^2+\sqrt{1-\frac{c^2d^2}{e^2}}\,e\,e^{-\text{ArcTanh}\left[\frac{c\,d}{e}\right]}\text{ArcTanh}[c\,x]^2+$$
$$2\,c\,d\,\text{ArcTanh}[c\,x]\,\text{Log}\left[1-e^{-2\,\text{ArcTanh}[c\,x]}\right]+i\,c\,d\,\pi\,\text{Log}\left[1+e^{2\,\text{ArcTanh}[c\,x]}\right]-2\,c\,d\,\text{ArcTanh}\left[\frac{c\,d}{e}\right]$$
$$\text{Log}\left[1-e^{-2\left(\text{ArcTanh}\left[\frac{c\,d}{e}\right]+\text{ArcTanh}[c\,x]\right)}\right]-2\,c\,d\,\text{ArcTanh}[c\,x]\,\text{Log}\left[1-e^{-2\left(\text{ArcTanh}\left[\frac{c\,d}{e}\right]+\text{ArcTanh}[c\,x]\right)}\right]+$$
$$\frac{1}{2}i\,c\,d\,\pi\,\text{Log}\left[1-c^2x^2\right]+2\,c\,d\,\text{ArcTanh}\left[\frac{c\,d}{e}\right]\text{Log}\left[i\,\text{Sinh}\left[\text{ArcTanh}\left[\frac{c\,d}{e}\right]+\text{ArcTanh}[c\,x]\right]\right]-$$
$$\left.\left.c\,d\,\text{PolyLog}\left[2,e^{-2\,\text{ArcTanh}[c\,x]}\right]+c\,d\,\text{PolyLog}\left[2,e^{-2\left(\text{ArcTanh}\left[\frac{c\,d}{e}\right]+\text{ArcTanh}[c\,x]\right)}\right]\right)\right)$$

Problem 152: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{a+b\,\text{ArcTanh}[c\,x]}{x^2\,(d+e\,x)}\,dx$$

Optimal (type 4, 200 leaves, 12 steps):

$$-\frac{a+b\,\text{ArcTanh}[c\,x]}{d\,x}+\frac{b\,c\,\text{Log}[x]}{d}-\frac{a\,e\,\text{Log}[x]}{d^2}-\frac{e\,(a+b\,\text{ArcTanh}[c\,x])\,\text{Log}\left[\frac{2}{1+c\,x}\right]}{d^2}+\frac{e\,(a+b\,\text{ArcTanh}[c\,x])\,\text{Log}\left[\frac{2\,c\,(d+e\,x)}{(c\,d+e)\,(1+c\,x)}\right]}{d^2}-\frac{b\,c\,\text{Log}\left[1-c^2\,x^2\right]}{2\,d}+\frac{b\,e\,\text{PolyLog}[2,-c\,x]}{2\,d^2}-\frac{b\,e\,\text{PolyLog}[2,c\,x]}{2\,d^2}+\frac{b\,e\,\text{PolyLog}\left[2,1-\frac{2}{1+c\,x}\right]}{2\,d^2}-\frac{b\,e\,\text{PolyLog}\left[2,1-\frac{2\,c\,(d+e\,x)}{(c\,d+e)\,(1+c\,x)}\right]}{2\,d^2}$$

Result (type 4, 360 leaves):

$$-\frac{1}{2\,d^3}\left(\frac{2\,a\,d^2}{x}-i\,b\,d\,e\,\pi\,\text{ArcTanh}[c\,x]+\frac{2\,b\,d^2\,\text{ArcTanh}[c\,x]}{x}-2\,b\,d\,e\,\text{ArcTanh}\left[\frac{c\,d}{e}\right]\,\text{ArcTanh}[c\,x]+b\,d\,e\,\text{ArcTanh}[c\,x]^2-\frac{b\,e^2\,\text{ArcTanh}[c\,x]^2}{c}+\frac{b\,\sqrt{1-\frac{c^2\,d^2}{e^2}}\,e^2\,e^{-\text{ArcTanh}\left[\frac{c\,d}{e}\right]}\,\text{ArcTanh}[c\,x]^2}{c}+2\,b\,d\,e\,\text{ArcTanh}[c\,x]\,\text{Log}\left[1-e^{-2\,\text{ArcTanh}[c\,x]}\right]+i\,b\,d\,e\,\pi\,\text{Log}\left[1+e^{2\,\text{ArcTanh}[c\,x]}\right]-2\,b\,d\,e\,\text{ArcTanh}\left[\frac{c\,d}{e}\right]\,\text{Log}\left[1-e^{-2\,\left(\text{ArcTanh}\left[\frac{c\,d}{e}\right]+\text{ArcTanh}[c\,x]\right)}\right]-2\,b\,d\,e\,\text{ArcTanh}[c\,x]\,\text{Log}\left[1-e^{-2\,\left(\text{ArcTanh}\left[\frac{c\,d}{e}\right]+\text{ArcTanh}[c\,x]\right)}\right]+2\,a\,d\,e\,\text{Log}[x]-2\,a\,d\,e\,\text{Log}[d+e\,x]-2\,b\,c\,d^2\,\text{Log}\left[\frac{c\,x}{\sqrt{1-c^2\,x^2}}\right]+\frac{1}{2}\,i\,b\,d\,e\,\pi\,\text{Log}\left[1-c^2\,x^2\right]+2\,b\,d\,e\,\text{ArcTanh}\left[\frac{c\,d}{e}\right]\,\text{Log}\left[i\,\text{Sinh}\left[\text{ArcTanh}\left[\frac{c\,d}{e}\right]+\text{ArcTanh}[c\,x]\right]\right]-b\,d\,e\,\text{PolyLog}\left[2,e^{-2\,\text{ArcTanh}[c\,x]}\right]+b\,d\,e\,\text{PolyLog}\left[2,e^{-2\,\left(\text{ArcTanh}\left[\frac{c\,d}{e}\right]+\text{ArcTanh}[c\,x]\right)}\right]\right)$$

Problem 153: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{a+b\,\text{ArcTanh}[c\,x]}{x^3\,(d+e\,x)}\,dx$$

Optimal (type 4, 261 leaves, 15 steps):

$$-\frac{b c}{2 d x}+\frac{b c^2 \text{ArcTanh}[c x]}{2 d}-\frac{a+b \text{ArcTanh}[c x]}{2 d x^2}+\frac{e (a+b \text{ArcTanh}[c x])}{d^2 x}-\frac{b c e \text{Log}[x]}{d^2}+\frac{a e^2 \text{Log}[x]}{d^3}+\frac{e^2 (a+b \text{ArcTanh}[c x]) \text{Log}\left[\frac{2}{1+c x}\right]}{d^3}-\frac{e^2 (a+b \text{ArcTanh}[c x]) \text{Log}\left[\frac{2 c (d+e x)}{(c d+e) (1+c x)}\right]}{d^3}+\frac{b c e \text{Log}\left[1-c^2 x^2\right]}{2 d^2}-\frac{b e^2 \text{PolyLog}[2,-c x]}{2 d^3}+\frac{b e^2 \text{PolyLog}[2,c x]}{2 d^3}-\frac{b e^2 \text{PolyLog}\left[2,1-\frac{2}{1+c x}\right]}{2 d^3}+\frac{b e^2 \text{PolyLog}\left[2,1-\frac{2 c (d+e x)}{(c d+e) (1+c x)}\right]}{2 d^3}$$

Result (type 4, 435 leaves):

$$\frac{1}{4 d^4}\left(-\frac{2 a d^3}{x^2}-\frac{2 b c d^3}{x}+\frac{4 a d^2 e}{x}+2 b c^2 d^3 \text{ArcTanh}[c x]-2 i b d e^2 \pi \text{ArcTanh}[c x]-\frac{2 b d^3 \text{ArcTanh}[c x]}{x^2}+\frac{4 b d^2 e \text{ArcTanh}[c x]}{x}-4 b d e^2 \text{ArcTanh}\left[\frac{c d}{e}\right] \text{ArcTanh}[c x]+2 b d e^2 \text{ArcTanh}[c x]^2-\frac{2 b e^3 \text{ArcTanh}[c x]^2}{c}+\frac{2 b \sqrt{1-\frac{c^2 d^2}{e^2}}\, e^3 e^{-\text{ArcTanh}\left[\frac{c d}{e}\right]} \text{ArcTanh}[c x]^2}{c}+4 b d e^2 \text{ArcTanh}[c x] \text{Log}\left[1-e^{-2 \text{ArcTanh}[c x]}\right]+2 i b d e^2 \pi \text{Log}\left[1+e^{2 \text{ArcTanh}[c x]}\right]-4 b d e^2 \text{ArcTanh}\left[\frac{c d}{e}\right] \text{Log}\left[1-e^{-2 \left(\text{ArcTanh}\left[\frac{c d}{e}\right]+\text{ArcTanh}[c x]\right)}\right]-4 b d e^2 \text{ArcTanh}[c x] \text{Log}\left[1-e^{-2 \left(\text{ArcTanh}\left[\frac{c d}{e}\right]+\text{ArcTanh}[c x]\right)}\right]+4 a d e^2 \text{Log}[x]-4 a d e^2 \text{Log}[d+e x]-4 b c d^2 e \text{Log}\left[\frac{c x}{\sqrt{1-c^2 x^2}}\right]+i b d e^2 \pi \text{Log}\left[1-c^2 x^2\right]+4 b d e^2 \text{ArcTanh}\left[\frac{c d}{e}\right] \text{Log}\left[i \text{Sinh}\left[\text{ArcTanh}\left[\frac{c d}{e}\right]+\text{ArcTanh}[c x]\right]\right]-2 b d e^2 \text{PolyLog}\left[2,e^{-2 \text{ArcTanh}[c x]}\right]+2 b d e^2 \text{PolyLog}\left[2,e^{-2 \left(\text{ArcTanh}\left[\frac{c d}{e}\right]+\text{ArcTanh}[c x]\right)}\right]\right)$$

Problem 154: Unable to integrate problem.

$$\int \frac{x^2 (a+b \text{ArcTanh}[c x])^2}{d+e x} \, dx$$

Optimal (type 4, 385 leaves, 14 steps):

$$\frac{a\,b\,x}{c\,e}+\frac{b^2\,x\,\text{ArcTanh}[c\,x]}{c\,e}-\frac{d\,(a+b\,\text{ArcTanh}[c\,x])^2}{c\,e^2}-\frac{(a+b\,\text{ArcTanh}[c\,x])^2}{2\,c^2\,e}-\frac{d\,x\,(a+b\,\text{ArcTanh}[c\,x])^2}{e^2}+\frac{x^2\,(a+b\,\text{ArcTanh}[c\,x])^2}{2\,e}+\frac{2\,b\,d\,(a+b\,\text{ArcTanh}[c\,x])\,\text{Log}\left[\frac{2}{1-c\,x}\right]}{c\,e^2}-\frac{d^2\,(a+b\,\text{ArcTanh}[c\,x])^2\,\text{Log}\left[\frac{2}{1+c\,x}\right]}{e^3}+\frac{d^2\,(a+b\,\text{ArcTanh}[c\,x])^2\,\text{Log}\left[\frac{2\,c\,(d+e\,x)}{(c\,d+e)\,(1+c\,x)}\right]}{e^3}+\frac{b^2\,\text{Log}\left[1-c^2\,x^2\right]}{2\,c^2\,e}+\frac{b^2\,d\,\text{PolyLog}\left[2,1-\frac{2}{1-c\,x}\right]}{c\,e^2}+\frac{b\,d^2\,(a+b\,\text{ArcTanh}[c\,x])\,\text{PolyLog}\left[2,1-\frac{2}{1+c\,x}\right]}{e^3}-\frac{b\,d^2\,(a+b\,\text{ArcTanh}[c\,x])\,\text{PolyLog}\left[2,1-\frac{2\,c\,(d+e\,x)}{(c\,d+e)\,(1+c\,x)}\right]}{e^3}+\frac{b^2\,d^2\,\text{PolyLog}\left[3,1-\frac{2}{1+c\,x}\right]}{2\,e^3}-\frac{b^2\,d^2\,\text{PolyLog}\left[3,1-\frac{2\,c\,(d+e\,x)}{(c\,d+e)\,(1+c\,x)}\right]}{2\,e^3}$$

Result (type 8, 23 leaves):

$$\int\frac{x^2\,(a+b\,\text{ArcTanh}[c\,x])^2}{d+e\,x}\,dx$$

Problem 155: Unable to integrate problem.

$$\int\frac{x\,(a+b\,\text{ArcTanh}[c\,x])^2}{d+e\,x}\,dx$$

Optimal (type 4, 279 leaves, 8 steps):

$$\frac{(a+b\,\text{ArcTanh}[c\,x])^2}{c\,e}+\frac{x\,(a+b\,\text{ArcTanh}[c\,x])^2}{e}-\frac{2\,b\,(a+b\,\text{ArcTanh}[c\,x])\,\text{Log}\left[\frac{2}{1-c\,x}\right]}{c\,e}+\frac{d\,(a+b\,\text{ArcTanh}[c\,x])^2\,\text{Log}\left[\frac{2}{1+c\,x}\right]}{e^2}-\frac{d\,(a+b\,\text{ArcTanh}[c\,x])^2\,\text{Log}\left[\frac{2\,c\,(d+e\,x)}{(c\,d+e)\,(1+c\,x)}\right]}{e^2}-\frac{b^2\,\text{PolyLog}\left[2,1-\frac{2}{1-c\,x}\right]}{c\,e}-\frac{b\,d\,(a+b\,\text{ArcTanh}[c\,x])\,\text{PolyLog}\left[2,1-\frac{2}{1+c\,x}\right]}{e^2}+\frac{b\,d\,(a+b\,\text{ArcTanh}[c\,x])\,\text{PolyLog}\left[2,1-\frac{2\,c\,(d+e\,x)}{(c\,d+e)\,(1+c\,x)}\right]}{e^2}-\frac{b^2\,d\,\text{PolyLog}\left[3,1-\frac{2}{1+c\,x}\right]}{2\,e^2}+\frac{b^2\,d\,\text{PolyLog}\left[3,1-\frac{2\,c\,(d+e\,x)}{(c\,d+e)\,(1+c\,x)}\right]}{2\,e^2}$$

Result (type 8, 21 leaves):

$$\int\frac{x\,(a+b\,\text{ArcTanh}[c\,x])^2}{d+e\,x}\,dx$$

Problem 156: Unable to integrate problem.

$$\int \frac{(a+b\,\text{ArcTanh}[c\,x])^2}{d+e\,x}\,dx$$

Optimal (type 4, 188 leaves, 1 step):

$$-\frac{(a+b\,\text{ArcTanh}[c\,x])^2\,\text{Log}\left[\frac{2}{1+c\,x}\right]}{e}+\frac{(a+b\,\text{ArcTanh}[c\,x])^2\,\text{Log}\left[\frac{2\,c\,(d+e\,x)}{(c\,d+e)\,(1+c\,x)}\right]}{e}+\frac{b\,(a+b\,\text{ArcTanh}[c\,x])\,\text{PolyLog}\left[2,1-\frac{2}{1+c\,x}\right]}{e}-\frac{b\,(a+b\,\text{ArcTanh}[c\,x])\,\text{PolyLog}\left[2,1-\frac{2\,c\,(d+e\,x)}{(c\,d+e)\,(1+c\,x)}\right]}{e}+\frac{b^2\,\text{PolyLog}\left[3,1-\frac{2}{1+c\,x}\right]}{2\,e}-\frac{b^2\,\text{PolyLog}\left[3,1-\frac{2\,c\,(d+e\,x)}{(c\,d+e)\,(1+c\,x)}\right]}{2\,e}$$

Result (type 8, 20 leaves):

$$\int \frac{(a+b\,\text{ArcTanh}[c\,x])^2}{d+e\,x}\,dx$$

Problem 157: Unable to integrate problem.

$$\int \frac{(a+b\,\text{ArcTanh}[c\,x])^2}{x\,(d+e\,x)}\,dx$$

Optimal (type 4, 319 leaves, 9 steps):

$$\frac{2\,(a+b\,\text{ArcTanh}[c\,x])^2\,\text{ArcTanh}\left[1-\frac{2}{1-c\,x}\right]}{d}+\frac{(a+b\,\text{ArcTanh}[c\,x])^2\,\text{Log}\left[\frac{2}{1+c\,x}\right]}{d}-\frac{(a+b\,\text{ArcTanh}[c\,x])^2\,\text{Log}\left[\frac{2\,c\,(d+e\,x)}{(c\,d+e)\,(1+c\,x)}\right]}{d}-\frac{b\,(a+b\,\text{ArcTanh}[c\,x])\,\text{PolyLog}\left[2,1-\frac{2}{1-c\,x}\right]}{d}+\frac{b\,(a+b\,\text{ArcTanh}[c\,x])\,\text{PolyLog}\left[2,-1+\frac{2}{1-c\,x}\right]}{d}-\frac{b\,(a+b\,\text{ArcTanh}[c\,x])\,\text{PolyLog}\left[2,1-\frac{2}{1+c\,x}\right]}{d}+\frac{b\,(a+b\,\text{ArcTanh}[c\,x])\,\text{PolyLog}\left[2,1-\frac{2\,c\,(d+e\,x)}{(c\,d+e)\,(1+c\,x)}\right]}{d}+\frac{b^2\,\text{PolyLog}\left[3,1-\frac{2}{1-c\,x}\right]}{2\,d}-\frac{b^2\,\text{PolyLog}\left[3,-1+\frac{2}{1-c\,x}\right]}{2\,d}-\frac{b^2\,\text{PolyLog}\left[3,1-\frac{2}{1+c\,x}\right]}{2\,d}+\frac{b^2\,\text{PolyLog}\left[3,1-\frac{2\,c\,(d+e\,x)}{(c\,d+e)\,(1+c\,x)}\right]}{2\,d}$$

Result (type 8, 23 leaves):

$$\int \frac{(a+b\,\text{ArcTanh}[c\,x])^2}{x\,(d+e\,x)}\,dx$$

Problem 158: Unable to integrate problem.

$$\int \frac{(a + b \operatorname{ArcTanh}[c x])^2}{x^2 (d + e x)} \, dx$$

Optimal (type 4, 412 leaves, 13 steps):

$$\frac{c (a + b \operatorname{ArcTanh}[c x])^2}{d} - \frac{(a + b \operatorname{ArcTanh}[c x])^2}{d x} - \frac{2 e (a + b \operatorname{ArcTanh}[c x])^2 \operatorname{ArcTanh}\left[1 - \frac{2}{1-c x}\right]}{d^2} - \frac{e (a + b \operatorname{ArcTanh}[c x])^2 \operatorname{Log}\left[\frac{2}{1+c x}\right]}{d^2} + \frac{e (a + b \operatorname{ArcTanh}[c x])^2 \operatorname{Log}\left[\frac{2 c (d+e x)}{(c d+e) (1+c x)}\right]}{d^2} + \frac{2 b c (a + b \operatorname{ArcTanh}[c x]) \operatorname{Log}\left[2 - \frac{2}{1+c x}\right]}{d} + \frac{b e (a + b \operatorname{ArcTanh}[c x]) \operatorname{PolyLog}\left[2, 1 - \frac{2}{1-c x}\right]}{d^2} - \frac{b e (a + b \operatorname{ArcTanh}[c x]) \operatorname{PolyLog}\left[2, -1 + \frac{2}{1-c x}\right]}{d^2} + \frac{b e (a + b \operatorname{ArcTanh}[c x]) \operatorname{PolyLog}\left[2, 1 - \frac{2}{1+c x}\right]}{d^2} - \frac{b^2 c \operatorname{PolyLog}\left[2, -1 + \frac{2}{1+c x}\right]}{d} - \frac{b e (a + b \operatorname{ArcTanh}[c x]) \operatorname{PolyLog}\left[2, 1 - \frac{2 c (d+e x)}{(c d+e) (1+c x)}\right]}{d^2} - \frac{b^2 e \operatorname{PolyLog}\left[3, 1 - \frac{2}{1-c x}\right]}{2 d^2} + \frac{b^2 e \operatorname{PolyLog}\left[3, -1 + \frac{2}{1-c x}\right]}{2 d^2} + \frac{b^2 e \operatorname{PolyLog}\left[3, 1 - \frac{2}{1+c x}\right]}{2 d^2} - \frac{b^2 e \operatorname{PolyLog}\left[3, 1 - \frac{2 c (d+e x)}{(c d+e) (1+c x)}\right]}{2 d^2}$$

Result (type 8, 23 leaves):

$$\int \frac{(a + b \operatorname{ArcTanh}[c x])^2}{x^2 (d + e x)} \, dx$$

Problem 159: Unable to integrate problem.

$$\int \frac{\operatorname{ArcTanh}[c x]^2}{x (d + e x)} \, dx$$

Optimal (type 4, 275 leaves, 9 steps):

$$\frac{2\,\text{ArcTanh}[c\,x]^2\,\text{ArcTanh}\left[1-\frac{2}{1-c\,x}\right]}{d}+\frac{\text{ArcTanh}[c\,x]^2\,\text{Log}\left[\frac{2}{1+c\,x}\right]}{d}-\frac{\text{ArcTanh}[c\,x]^2\,\text{Log}\left[\frac{2\,c\,(d+e\,x)}{(c\,d+e)\,(1+c\,x)}\right]}{d}-\frac{\text{ArcTanh}[c\,x]\,\text{PolyLog}\left[2,1-\frac{2}{1-c\,x}\right]}{d}+\frac{\text{ArcTanh}[c\,x]\,\text{PolyLog}\left[2,-1+\frac{2}{1-c\,x}\right]}{d}-\frac{\text{ArcTanh}[c\,x]\,\text{PolyLog}\left[2,1-\frac{2}{1+c\,x}\right]}{d}+\frac{\text{ArcTanh}[c\,x]\,\text{PolyLog}\left[2,1-\frac{2\,c\,(d+e\,x)}{(c\,d+e)\,(1+c\,x)}\right]}{d}+\frac{\text{PolyLog}\left[3,1-\frac{2}{1-c\,x}\right]}{2\,d}-\frac{\text{PolyLog}\left[3,-1+\frac{2}{1-c\,x}\right]}{2\,d}-\frac{\text{PolyLog}\left[3,1-\frac{2}{1+c\,x}\right]}{2\,d}+\frac{\text{PolyLog}\left[3,1-\frac{2\,c\,(d+e\,x)}{(c\,d+e)\,(1+c\,x)}\right]}{2\,d}$$

Result (type 8, 19 leaves):

$$\int\frac{\text{ArcTanh}[c\,x]^2}{x\,(d+e\,x)}\,dx$$

Problem 212: Result unnecessarily involves imaginary or complex numbers.

$$\int\frac{\left(1-a^2\,x^2\right)^2\,\text{ArcTanh}[a\,x]^2}{x^5}\,dx$$

Optimal (type 4, 214 leaves, 29 steps):

$$-\frac{a^2}{12\,x^2}-\frac{a\,\text{ArcTanh}[a\,x]}{6\,x^3}+\frac{3\,a^3\,\text{ArcTanh}[a\,x]}{2\,x}-\frac{3}{4}\,a^4\,\text{ArcTanh}[a\,x]^2-\frac{\text{ArcTanh}[a\,x]^2}{4\,x^4}+\frac{a^2\,\text{ArcTanh}[a\,x]^2}{x^2}+2\,a^4\,\text{ArcTanh}[a\,x]^2\,\text{ArcTanh}\left[1-\frac{2}{1-a\,x}\right]-\frac{4}{3}\,a^4\,\text{Log}[x]+\frac{2}{3}\,a^4\,\text{Log}\left[1-a^2\,x^2\right]-a^4\,\text{ArcTanh}[a\,x]\,\text{PolyLog}\left[2,1-\frac{2}{1-a\,x}\right]+a^4\,\text{ArcTanh}[a\,x]\,\text{PolyLog}\left[2,-1+\frac{2}{1-a\,x}\right]+\frac{1}{2}\,a^4\,\text{PolyLog}\left[3,1-\frac{2}{1-a\,x}\right]-\frac{1}{2}\,a^4\,\text{PolyLog}\left[3,-1+\frac{2}{1-a\,x}\right]$$

Result (type 4, 238 leaves):

$$\frac{1}{24}\left(2\,a^4+i\,a^4\,\pi^3-\frac{2\,a^2}{x^2}-\frac{4\,a\,\text{ArcTanh}[a\,x]}{x^3}+\frac{36\,a^3\,\text{ArcTanh}[a\,x]}{x}-18\,a^4\,\text{ArcTanh}[a\,x]^2-\frac{6\,\text{ArcTanh}[a\,x]^2}{x^4}+\frac{24\,a^2\,\text{ArcTanh}[a\,x]^2}{x^2}-16\,a^4\,\text{ArcTanh}[a\,x]^3-24\,a^4\,\text{ArcTanh}[a\,x]^2\,\text{Log}\left[1+e^{-2\,\text{ArcTanh}[a\,x]}\right]+24\,a^4\,\text{ArcTanh}[a\,x]^2\,\text{Log}\left[1-e^{2\,\text{ArcTanh}[a\,x]}\right]-32\,a^4\,\text{Log}\left[\frac{a\,x}{\sqrt{1-a^2\,x^2}}\right]+24\,a^4\,\text{ArcTanh}[a\,x]\,\text{PolyLog}\left[2,-e^{-2\,\text{ArcTanh}[a\,x]}\right]+24\,a^4\,\text{ArcTanh}[a\,x]\,\text{PolyLog}\left[2,e^{2\,\text{ArcTanh}[a\,x]}\right]+12\,a^4\,\text{PolyLog}\left[3,-e^{-2\,\text{ArcTanh}[a\,x]}\right]-12\,a^4\,\text{PolyLog}\left[3,e^{2\,\text{ArcTanh}[a\,x]}\right]\right)$$

Problem 240: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{\text{ArcTanh}[a\,x]^2}{x^3\,(1-a^2\,x^2)}\,dx$$

Optimal (type 4, 138 leaves, 13 steps):

$$-\frac{a\,\text{ArcTanh}[a\,x]}{x}+\frac{1}{2}\,a^2\,\text{ArcTanh}[a\,x]^2-\frac{\text{ArcTanh}[a\,x]^2}{2\,x^2}+\frac{1}{3}\,a^2\,\text{ArcTanh}[a\,x]^3+a^2\,\text{Log}[x]-\frac{1}{2}\,a^2\,\text{Log}\left[1-a^2\,x^2\right]+a^2\,\text{ArcTanh}[a\,x]^2\,\text{Log}\left[2-\frac{2}{1+a\,x}\right]-a^2\,\text{ArcTanh}[a\,x]\,\text{PolyLog}\left[2,-1+\frac{2}{1+a\,x}\right]-\frac{1}{2}\,a^2\,\text{PolyLog}\left[3,-1+\frac{2}{1+a\,x}\right]$$

Result (type 4, 133 leaves):

$$-a^2\left(-\frac{i\,\pi^3}{24}+\frac{\text{ArcTanh}[a\,x]}{a\,x}+\frac{(1-a^2\,x^2)\,\text{ArcTanh}[a\,x]^2}{2\,a^2\,x^2}+\frac{1}{3}\,\text{ArcTanh}[a\,x]^3-\text{ArcTanh}[a\,x]^2\,\text{Log}\left[1-e^{2\,\text{ArcTanh}[a\,x]}\right]-\text{Log}\left[\frac{a\,x}{\sqrt{1-a^2\,x^2}}\right]-\text{ArcTanh}[a\,x]\,\text{PolyLog}\left[2,e^{2\,\text{ArcTanh}[a\,x]}\right]+\frac{1}{2}\,\text{PolyLog}\left[3,e^{2\,\text{ArcTanh}[a\,x]}\right]\right)$$

Problem 246: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{\text{ArcTanh}[a\,x]^3}{x^2\,(1-a^2\,x^2)}\,dx$$

Optimal (type 4, 90 leaves, 7 steps):

$$a\,\text{ArcTanh}[a\,x]^3-\frac{\text{ArcTanh}[a\,x]^3}{x}+\frac{1}{4}\,a\,\text{ArcTanh}[a\,x]^4+3\,a\,\text{ArcTanh}[a\,x]^2\,\text{Log}\left[2-\frac{2}{1+a\,x}\right]-3\,a\,\text{ArcTanh}[a\,x]\,\text{PolyLog}\left[2,-1+\frac{2}{1+a\,x}\right]-\frac{3}{2}\,a\,\text{PolyLog}\left[3,-1+\frac{2}{1+a\,x}\right]$$

Result (type 4, 93 leaves):

$$-a\left(-\frac{i\,\pi^3}{8}+\text{ArcTanh}[a\,x]^3+\frac{\text{ArcTanh}[a\,x]^3}{a\,x}-\frac{1}{4}\,\text{ArcTanh}[a\,x]^4-3\,\text{ArcTanh}[a\,x]^2\,\text{Log}\left[1-e^{2\,\text{ArcTanh}[a\,x]}\right]-3\,\text{ArcTanh}[a\,x]\,\text{PolyLog}\left[2,e^{2\,\text{ArcTanh}[a\,x]}\right]+\frac{3}{2}\,\text{PolyLog}\left[3,e^{2\,\text{ArcTanh}[a\,x]}\right]\right)$$

Problem 270: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{\text{ArcTanh}[a\,x]^2}{x\,(1-a^2\,x^2)^2}\,dx$$

Optimal (type 4, 136 leaves, 8 steps):

$$\frac{1}{4\left(1-a^2x^2\right)}-\frac{a\,x\,\text{ArcTanh}[a\,x]}{2\left(1-a^2x^2\right)}-\frac{1}{4}\,\text{ArcTanh}[a\,x]^2+\frac{\text{ArcTanh}[a\,x]^2}{2\left(1-a^2x^2\right)}+\frac{1}{3}\,\text{ArcTanh}[a\,x]^3+\text{ArcTanh}[a\,x]^2\,\text{Log}\left[2-\frac{2}{1+a\,x}\right]-\text{ArcTanh}[a\,x]\,\text{PolyLog}\left[2,\,-1+\frac{2}{1+a\,x}\right]-\frac{1}{2}\,\text{PolyLog}\left[3,\,-1+\frac{2}{1+a\,x}\right]$$

Result (type 4, 106 leaves):

$$\frac{1}{24}\left(i\,\pi^3-8\,\text{ArcTanh}[a\,x]^3+3\,\text{Cosh}[2\,\text{ArcTanh}[a\,x]]+6\,\text{ArcTanh}[a\,x]^2\,\text{Cosh}[2\,\text{ArcTanh}[a\,x]]+24\,\text{ArcTanh}[a\,x]^2\,\text{Log}\left[1-e^{2\,\text{ArcTanh}[a\,x]}\right]+24\,\text{ArcTanh}[a\,x]\,\text{PolyLog}\left[2,\,e^{2\,\text{ArcTanh}[a\,x]}\right]-12\,\text{PolyLog}\left[3,\,e^{2\,\text{ArcTanh}[a\,x]}\right]-6\,\text{ArcTanh}[a\,x]\,\text{Sinh}[2\,\text{ArcTanh}[a\,x]]\right)$$

Problem 272: Result unnecessarily involves imaginary or complex numbers.

$$\int\frac{\text{ArcTanh}[a\,x]^2}{x^3\left(1-a^2x^2\right)^2}\,dx$$

Optimal (type 4, 205 leaves, 22 steps):

$$\frac{a^2}{4\left(1-a^2x^2\right)}-\frac{a\,\text{ArcTanh}[a\,x]}{x}-\frac{a^3\,x\,\text{ArcTanh}[a\,x]}{2\left(1-a^2x^2\right)}+\frac{1}{4}\,a^2\,\text{ArcTanh}[a\,x]^2-\frac{\text{ArcTanh}[a\,x]^2}{2\,x^2}+\frac{a^2\,\text{ArcTanh}[a\,x]^2}{2\left(1-a^2x^2\right)}+\frac{2}{3}\,a^2\,\text{ArcTanh}[a\,x]^3+a^2\,\text{Log}[x]-\frac{1}{2}\,a^2\,\text{Log}\left[1-a^2x^2\right]+2\,a^2\,\text{ArcTanh}[a\,x]^2\,\text{Log}\left[2-\frac{2}{1+a\,x}\right]-2\,a^2\,\text{ArcTanh}[a\,x]\,\text{PolyLog}\left[2,\,-1+\frac{2}{1+a\,x}\right]-a^2\,\text{PolyLog}\left[3,\,-1+\frac{2}{1+a\,x}\right]$$

Result (type 4, 146 leaves):

$$a^2\left(2\,\text{ArcTanh}[a\,x]\,\text{PolyLog}\left[2,\,e^{2\,\text{ArcTanh}[a\,x]}\right]+\frac{1}{24}\left(2\,i\,\pi^3-16\,\text{ArcTanh}[a\,x]^3+3\,\text{Cosh}[2\,\text{ArcTanh}[a\,x]]+6\,\text{ArcTanh}[a\,x]^2\left(2-\frac{2}{a^2x^2}+\text{Cosh}[2\,\text{ArcTanh}[a\,x]]+8\,\text{Log}\left[1-e^{2\,\text{ArcTanh}[a\,x]}\right]\right)+24\,\text{Log}\left[\frac{a\,x}{\sqrt{1-a^2x^2}}\right]-24\,\text{PolyLog}\left[3,\,e^{2\,\text{ArcTanh}[a\,x]}\right]-\frac{6\,\text{ArcTanh}[a\,x]\,(4+a\,x\,\text{Sinh}[2\,\text{ArcTanh}[a\,x]])}{a\,x}\right)\right)$$

Problem 278: Result unnecessarily involves imaginary or complex numbers.

$$\int\frac{\text{ArcTanh}[a\,x]^3}{x^2\left(1-a^2x^2\right)^2}\,dx$$

Optimal (type 4, 191 leaves, 12 steps):

$$-\frac{3a}{8\left(1-a^2x^2\right)}+\frac{3a^2x\,\text{ArcTanh}[a\,x]}{4\left(1-a^2x^2\right)}+\frac{3}{8}a\,\text{ArcTanh}[a\,x]^2-\frac{3a\,\text{ArcTanh}[a\,x]^2}{4\left(1-a^2x^2\right)}+a\,\text{ArcTanh}[a\,x]^3-\frac{\text{ArcTanh}[a\,x]^3}{x}+\frac{a^2x\,\text{ArcTanh}[a\,x]^3}{2\left(1-a^2x^2\right)}+\frac{3}{8}a\,\text{ArcTanh}[a\,x]^4+3a\,\text{ArcTanh}[a\,x]^2\,\text{Log}\left[2-\frac{2}{1+a\,x}\right]-3a\,\text{ArcTanh}[a\,x]\,\text{PolyLog}\left[2,-1+\frac{2}{1+a\,x}\right]-\frac{3}{2}a\,\text{PolyLog}\left[3,-1+\frac{2}{1+a\,x}\right]$$

Result (type 4, 144 leaves):

$$\frac{1}{16}a\left(2\,i\,\pi^3-16\,\text{ArcTanh}[a\,x]^3-\frac{16\,\text{ArcTanh}[a\,x]^3}{a\,x}+6\,\text{ArcTanh}[a\,x]^4-3\,\text{Cosh}[2\,\text{ArcTanh}[a\,x]]-6\,\text{ArcTanh}[a\,x]^2\,\text{Cosh}[2\,\text{ArcTanh}[a\,x]]+48\,\text{ArcTanh}[a\,x]^2\,\text{Log}\left[1-e^{2\,\text{ArcTanh}[a\,x]}\right]+48\,\text{ArcTanh}[a\,x]\,\text{PolyLog}\left[2,e^{2\,\text{ArcTanh}[a\,x]}\right]-24\,\text{PolyLog}\left[3,e^{2\,\text{ArcTanh}[a\,x]}\right]+6\,\text{ArcTanh}[a\,x]\,\text{Sinh}[2\,\text{ArcTanh}[a\,x]]+4\,\text{ArcTanh}[a\,x]^3\,\text{Sinh}[2\,\text{ArcTanh}[a\,x]]\right)$$

Problem 282: Attempted integration timed out after 120 seconds.

$$\int\frac{x^3}{\left(1-a^2x^2\right)^2\text{ArcTanh}[a\,x]}\,dx$$

Optimal (type 8, 43 leaves, 0 steps):

$$\frac{\text{SinhIntegral}[2\,\text{ArcTanh}[a\,x]]}{2a^4}-\frac{\text{Int}\left[\frac{x}{\left(1-a^2x^2\right)\text{ArcTanh}[a\,x]},x\right]}{a^2}$$

Result (type 1, 1 leaves):

???

Problem 312: Result unnecessarily involves imaginary or complex numbers.

$$\int\frac{\text{ArcTanh}[a\,x]^2}{x\left(1-a^2x^2\right)^3}\,dx$$

Optimal (type 4, 196 leaves, 13 steps):

$$\frac{1}{32\left(1-a^2x^2\right)^2}+\frac{11}{32\left(1-a^2x^2\right)}-\frac{a\,x\,\text{ArcTanh}[a\,x]}{8\left(1-a^2x^2\right)^2}-\frac{11\,a\,x\,\text{ArcTanh}[a\,x]}{16\left(1-a^2x^2\right)}-\frac{11}{32}\text{ArcTanh}[a\,x]^2+\frac{\text{ArcTanh}[a\,x]^2}{4\left(1-a^2x^2\right)^2}+\frac{\text{ArcTanh}[a\,x]^2}{2\left(1-a^2x^2\right)}+\frac{1}{3}\text{ArcTanh}[a\,x]^3+\text{ArcTanh}[a\,x]^2\,\text{Log}\left[2-\frac{2}{1+a\,x}\right]-\text{ArcTanh}[a\,x]\,\text{PolyLog}\left[2,-1+\frac{2}{1+a\,x}\right]-\frac{1}{2}\text{PolyLog}\left[3,-1+\frac{2}{1+a\,x}\right]$$

Result (type 4, 129 leaves):

$$\text{ArcTanh}[a\,x]\,\text{PolyLog}\left[2,\,e^{2\,\text{ArcTanh}[a\,x]}\right]+\frac{1}{768}\left(32\,i\,\pi^3-256\,\text{ArcTanh}[a\,x]^3+144\,\text{Cosh}[2\,\text{ArcTanh}[a\,x]]+3\,\text{Cosh}[4\,\text{ArcTanh}[a\,x]]+24\,\text{ArcTanh}[a\,x]^2\left(12\,\text{Cosh}[2\,\text{ArcTanh}[a\,x]]+\text{Cosh}[4\,\text{ArcTanh}[a\,x]]+32\,\text{Log}\left[1-e^{2\,\text{ArcTanh}[a\,x]}\right]\right)-384\,\text{PolyLog}\left[3,\,e^{2\,\text{ArcTanh}[a\,x]}\right]-12\,\text{ArcTanh}[a\,x]\left(24\,\text{Sinh}[2\,\text{ArcTanh}[a\,x]]+\text{Sinh}[4\,\text{ArcTanh}[a\,x]]\right)\right)$$

Problem 319: Result unnecessarily involves imaginary or complex numbers.

$$\int\frac{\text{ArcTanh}[a\,x]^3}{x^2\left(1-a^2\,x^2\right)^3}\,dx$$

Optimal (type 4, 281 leaves, 21 steps):

$$-\frac{3\,a}{128\left(1-a^2\,x^2\right)^2}-\frac{93\,a}{128\left(1-a^2\,x^2\right)}+\frac{3\,a^2\,x\,\text{ArcTanh}[a\,x]}{32\left(1-a^2\,x^2\right)^2}+\frac{93\,a^2\,x\,\text{ArcTanh}[a\,x]}{64\left(1-a^2\,x^2\right)}+\frac{93}{128}\,a\,\text{ArcTanh}[a\,x]^2-\frac{3\,a\,\text{ArcTanh}[a\,x]^2}{16\left(1-a^2\,x^2\right)^2}-\frac{21\,a\,\text{ArcTanh}[a\,x]^2}{16\left(1-a^2\,x^2\right)}+a\,\text{ArcTanh}[a\,x]^3-\frac{\text{ArcTanh}[a\,x]^3}{x}+\frac{a^2\,x\,\text{ArcTanh}[a\,x]^3}{4\left(1-a^2\,x^2\right)^2}+\frac{7\,a^2\,x\,\text{ArcTanh}[a\,x]^3}{8\left(1-a^2\,x^2\right)}+\frac{15}{32}\,a\,\text{ArcTanh}[a\,x]^4+3\,a\,\text{ArcTanh}[a\,x]^2\,\text{Log}\left[2-\frac{2}{1+a\,x}\right]-3\,a\,\text{ArcTanh}[a\,x]\,\text{PolyLog}\left[2,\,-1+\frac{2}{1+a\,x}\right]-\frac{3}{2}\,a\,\text{PolyLog}\left[3,\,-1+\frac{2}{1+a\,x}\right]$$

Result (type 4, 218 leaves):

$$-a\left(-\frac{i\,\pi^3}{8}+\text{ArcTanh}[a\,x]^3+\frac{\text{ArcTanh}[a\,x]^3}{a\,x}-\frac{a\,x\,\text{ArcTanh}[a\,x]^3}{1-a^2\,x^2}-\frac{15}{32}\,\text{ArcTanh}[a\,x]^4+\frac{3}{8}\,\text{Cosh}[2\,\text{ArcTanh}[a\,x]]+\frac{3}{4}\,\text{ArcTanh}[a\,x]^2\,\text{Cosh}[2\,\text{ArcTanh}[a\,x]]+\frac{3\,\text{Cosh}[4\,\text{ArcTanh}[a\,x]]}{1024}+\frac{3}{128}\,\text{ArcTanh}[a\,x]^2\,\text{Cosh}[4\,\text{ArcTanh}[a\,x]]-3\,\text{ArcTanh}[a\,x]^2\,\text{Log}\left[1-e^{2\,\text{ArcTanh}[a\,x]}\right]-3\,\text{ArcTanh}[a\,x]\,\text{PolyLog}\left[2,\,e^{2\,\text{ArcTanh}[a\,x]}\right]+\frac{3}{2}\,\text{PolyLog}\left[3,\,e^{2\,\text{ArcTanh}[a\,x]}\right]-\frac{3}{4}\,\text{ArcTanh}[a\,x]\,\text{Sinh}[2\,\text{ArcTanh}[a\,x]]-\frac{3}{256}\,\text{ArcTanh}[a\,x]\,\text{Sinh}[4\,\text{ArcTanh}[a\,x]]-\frac{1}{32}\,\text{ArcTanh}[a\,x]^3\,\text{Sinh}[4\,\text{ArcTanh}[a\,x]]\right)$$

Problem 383: Result more than twice size of optimal antiderivative.

$$\int\frac{\text{ArcTanh}[a\,x]^3}{\sqrt{1-a^2\,x^2}}\,dx$$

Optimal (type 4, 153 leaves, 10 steps):

$$\frac{2\,\text{ArcTan}\left[e^{\text{ArcTanh}[a\,x]}\right]\,\text{ArcTanh}[a\,x]^3}{a}-\frac{3\,i\,\text{ArcTanh}[a\,x]^2\,\text{PolyLog}\left[2,-i\,e^{\text{ArcTanh}[a\,x]}\right]}{a}+\frac{3\,i\,\text{ArcTanh}[a\,x]^2\,\text{PolyLog}\left[2,i\,e^{\text{ArcTanh}[a\,x]}\right]}{a}+\frac{6\,i\,\text{ArcTanh}[a\,x]\,\text{PolyLog}\left[3,-i\,e^{\text{ArcTanh}[a\,x]}\right]}{a}-\frac{6\,i\,\text{ArcTanh}[a\,x]\,\text{PolyLog}\left[3,i\,e^{\text{ArcTanh}[a\,x]}\right]}{a}-\frac{6\,i\,\text{PolyLog}\left[4,-i\,e^{\text{ArcTanh}[a\,x]}\right]}{a}+\frac{6\,i\,\text{PolyLog}\left[4,i\,e^{\text{ArcTanh}[a\,x]}\right]}{a}$$

Result (type 4, 451 leaves):

$$-\frac{1}{64\,a}\,i\left(7\,\pi^4+8\,i\,\pi^3\,\text{ArcTanh}[a\,x]+24\,\pi^2\,\text{ArcTanh}[a\,x]^2-32\,i\,\pi\,\text{ArcTanh}[a\,x]^3-16\,\text{ArcTanh}[a\,x]^4+\right.$$
$$8\,i\,\pi^3\,\text{Log}\left[1+i\,e^{-\text{ArcTanh}[a\,x]}\right]+48\,\pi^2\,\text{ArcTanh}[a\,x]\,\text{Log}\left[1+i\,e^{-\text{ArcTanh}[a\,x]}\right]-$$
$$96\,i\,\pi\,\text{ArcTanh}[a\,x]^2\,\text{Log}\left[1+i\,e^{-\text{ArcTanh}[a\,x]}\right]-64\,\text{ArcTanh}[a\,x]^3\,\text{Log}\left[1+i\,e^{-\text{ArcTanh}[a\,x]}\right]-$$
$$48\,\pi^2\,\text{ArcTanh}[a\,x]\,\text{Log}\left[1-i\,e^{\text{ArcTanh}[a\,x]}\right]+96\,i\,\pi\,\text{ArcTanh}[a\,x]^2\,\text{Log}\left[1-i\,e^{\text{ArcTanh}[a\,x]}\right]-$$
$$8\,i\,\pi^3\,\text{Log}\left[1+i\,e^{\text{ArcTanh}[a\,x]}\right]+64\,\text{ArcTanh}[a\,x]^3\,\text{Log}\left[1+i\,e^{\text{ArcTanh}[a\,x]}\right]+$$
$$8\,i\,\pi^3\,\text{Log}\left[\text{Tan}\left[\frac{1}{4}\left(\pi+2\,i\,\text{ArcTanh}[a\,x]\right)\right]\right]-48\left(\pi-2\,i\,\text{ArcTanh}[a\,x]\right)^2$$
$$\text{PolyLog}\left[2,-i\,e^{-\text{ArcTanh}[a\,x]}\right]+192\,\text{ArcTanh}[a\,x]^2\,\text{PolyLog}\left[2,-i\,e^{\text{ArcTanh}[a\,x]}\right]-$$
$$48\,\pi^2\,\text{PolyLog}\left[2,i\,e^{\text{ArcTanh}[a\,x]}\right]+192\,i\,\pi\,\text{ArcTanh}[a\,x]\,\text{PolyLog}\left[2,i\,e^{\text{ArcTanh}[a\,x]}\right]+$$
$$192\,i\,\pi\,\text{PolyLog}\left[3,-i\,e^{-\text{ArcTanh}[a\,x]}\right]+384\,\text{ArcTanh}[a\,x]\,\text{PolyLog}\left[3,-i\,e^{-\text{ArcTanh}[a\,x]}\right]-$$
$$384\,\text{ArcTanh}[a\,x]\,\text{PolyLog}\left[3,-i\,e^{\text{ArcTanh}[a\,x]}\right]-192\,i\,\pi\,\text{PolyLog}\left[3,i\,e^{\text{ArcTanh}[a\,x]}\right]+$$
$$\left.384\,\text{PolyLog}\left[4,-i\,e^{-\text{ArcTanh}[a\,x]}\right]+384\,\text{PolyLog}\left[4,-i\,e^{\text{ArcTanh}[a\,x]}\right]\right)$$

Problem 405: Result more than twice size of optimal antiderivative.

$$\int\frac{x^2\,\text{ArcTanh}[a\,x]^3}{\left(1-a^2\,x^2\right)^{3/2}}\,dx$$

Optimal (type 4, 246 leaves, 13 steps):

$$-\frac{6}{a^3\sqrt{1-a^2\,x^2}}+\frac{6\,x\,\text{ArcTanh}[a\,x]}{a^2\sqrt{1-a^2\,x^2}}-\frac{3\,\text{ArcTanh}[a\,x]^2}{a^3\sqrt{1-a^2\,x^2}}+\frac{x\,\text{ArcTanh}[a\,x]^3}{a^2\sqrt{1-a^2\,x^2}}-\frac{2\,\text{ArcTan}\left[e^{\text{ArcTanh}[a\,x]}\right]\,\text{ArcTanh}[a\,x]^3}{a^3}+\frac{3\,i\,\text{ArcTanh}[a\,x]^2\,\text{PolyLog}\left[2,-i\,e^{\text{ArcTanh}[a\,x]}\right]}{a^3}-\frac{3\,i\,\text{ArcTanh}[a\,x]^2\,\text{PolyLog}\left[2,i\,e^{\text{ArcTanh}[a\,x]}\right]}{a^3}-\frac{6\,i\,\text{ArcTanh}[a\,x]\,\text{PolyLog}\left[3,-i\,e^{\text{ArcTanh}[a\,x]}\right]}{a^3}+\frac{6\,i\,\text{ArcTanh}[a\,x]\,\text{PolyLog}\left[3,i\,e^{\text{ArcTanh}[a\,x]}\right]}{a^3}+\frac{6\,i\,\text{PolyLog}\left[4,-i\,e^{\text{ArcTanh}[a\,x]}\right]}{a^3}-\frac{6\,i\,\text{PolyLog}\left[4,i\,e^{\text{ArcTanh}[a\,x]}\right]}{a^3}$$

Result (type 4, 541 leaves):

$$\frac{1}{64 a^3}\left(7 i \pi^4-\frac{384}{\sqrt{1-a^2 x^2}}-8 \pi^3 \text{ArcTanh}[a x]+\frac{384 a x \text{ArcTanh}[a x]}{\sqrt{1-a^2 x^2}}+\right.$$
$$24 i \pi^2 \text{ArcTanh}[a x]^2-\frac{192 \text{ArcTanh}[a x]^2}{\sqrt{1-a^2 x^2}}+32 \pi \text{ArcTanh}[a x]^3+$$
$$\frac{64 a x \text{ArcTanh}[a x]^3}{\sqrt{1-a^2 x^2}}-16 i \text{ArcTanh}[a x]^4-8 \pi^3 \text{Log}\left[1+i e^{-\text{ArcTanh}[a x]}\right]+$$
$$48 i \pi^2 \text{ArcTanh}[a x] \text{Log}\left[1+i e^{-\text{ArcTanh}[a x]}\right]+96 \pi \text{ArcTanh}[a x]^2 \text{Log}\left[1+i e^{-\text{ArcTanh}[a x]}\right]-$$
$$64 i \text{ArcTanh}[a x]^3 \text{Log}\left[1+i e^{-\text{ArcTanh}[a x]}\right]-48 i \pi^2 \text{ArcTanh}[a x] \text{Log}\left[1-i e^{\text{ArcTanh}[a x]}\right]-$$
$$96 \pi \text{ArcTanh}[a x]^2 \text{Log}\left[1-i e^{\text{ArcTanh}[a x]}\right]+8 \pi^3 \text{Log}\left[1+i e^{\text{ArcTanh}[a x]}\right]+$$
$$64 i \text{ArcTanh}[a x]^3 \text{Log}\left[1+i e^{\text{ArcTanh}[a x]}\right]-8 \pi^3 \text{Log}\left[\text{Tan}\left[\frac{1}{4}(\pi+2 i \text{ArcTanh}[a x])\right]\right]-$$
$$48 i (\pi-2 i \text{ArcTanh}[a x])^2 \text{PolyLog}\left[2,-i e^{-\text{ArcTanh}[a x]}\right]+$$
$$192 i \text{ArcTanh}[a x]^2 \text{PolyLog}\left[2,-i e^{\text{ArcTanh}[a x]}\right]-$$
$$48 i \pi^2 \text{PolyLog}\left[2, i e^{\text{ArcTanh}[a x]}\right]-192 \pi \text{ArcTanh}[a x] \text{PolyLog}\left[2, i e^{\text{ArcTanh}[a x]}\right]-$$
$$192 \pi \text{PolyLog}\left[3,-i e^{-\text{ArcTanh}[a x]}\right]+384 i \text{ArcTanh}[a x] \text{PolyLog}\left[3,-i e^{-\text{ArcTanh}[a x]}\right]-$$
$$384 i \text{ArcTanh}[a x] \text{PolyLog}\left[3,-i e^{\text{ArcTanh}[a x]}\right]+192 \pi \text{PolyLog}\left[3, i e^{\text{ArcTanh}[a x]}\right]+$$
$$\left.384 i \text{PolyLog}\left[4,-i e^{-\text{ArcTanh}[a x]}\right]+384 i \text{PolyLog}\left[4,-i e^{\text{ArcTanh}[a x]}\right]\right)$$

Problem 412: Attempted integration timed out after 120 seconds.

$$\int \frac{x^2}{\left(1-a^2 x^2\right)^{3/2} \text{ArcTanh}[a x]} \, dx$$

Optimal (type 8, 27 leaves, 0 steps):

$$\text{Int}\left[\frac{x^2}{\left(1-a^2 x^2\right)^{3/2} \text{ArcTanh}[a x]}, x\right]$$

Result (type 1, 1 leaves):

???

Problem 458: Result more than twice size of optimal antiderivative.

$$\int \frac{\left(1-a^2 x^2\right)^{3/2} \text{ArcTanh}[a x]}{x^7} \, dx$$

Optimal (type 4, 243 leaves, 24 steps):

$$-\frac{a \sqrt{1-a^2 x^2}}{30 x^5}+\frac{19 a^3 \sqrt{1-a^2 x^2}}{360 x^3}+\frac{31 a^5 \sqrt{1-a^2 x^2}}{720 x}-\frac{\sqrt{1-a^2 x^2} \text{ArcTanh}[a x]}{6 x^6}+$$
$$\frac{7 a^2 \sqrt{1-a^2 x^2} \text{ArcTanh}[a x]}{24 x^4}-\frac{a^4 \sqrt{1-a^2 x^2} \text{ArcTanh}[a x]}{16 x^2}-\frac{1}{8} a^6 \text{ArcTanh}[a x] \text{ArcTanh}\left[\frac{\sqrt{1-a x}}{\sqrt{1+a x}}\right]+$$
$$\frac{1}{16} a^6 \text{PolyLog}\left[2,-\frac{\sqrt{1-a x}}{\sqrt{1+a x}}\right]-\frac{1}{16} a^6 \text{PolyLog}\left[2, \frac{\sqrt{1-a x}}{\sqrt{1+a x}}\right]$$

Result (type 4, 530 leaves):

$$-\frac{1}{192} a^6 \left(-8 \operatorname{Coth}\left[\frac{1}{2} \operatorname{ArcTanh}[a x]\right]-6 \operatorname{ArcTanh}[a x] \operatorname{Csch}\left[\frac{1}{2} \operatorname{ArcTanh}[a x]\right]^2-\frac{a x \operatorname{Csch}\left[\frac{1}{2} \operatorname{ArcTanh}[a x]\right]^4}{\sqrt{1-a^2 x^2}}-3 \operatorname{ArcTanh}[a x] \operatorname{Csch}\left[\frac{1}{2} \operatorname{ArcTanh}[a x]\right]^4-24 \operatorname{ArcTanh}[a x] \operatorname{Log}\left[1-e^{-\operatorname{ArcTanh}[a x]}\right]+24 \operatorname{ArcTanh}[a x] \operatorname{Log}\left[1+e^{-\operatorname{ArcTanh}[a x]}\right]-24 \operatorname{PolyLog}\left[2,-e^{-\operatorname{ArcTanh}[a x]}\right]+24 \operatorname{PolyLog}\left[2, e^{-\operatorname{ArcTanh}[a x]}\right]-6 \operatorname{ArcTanh}[a x] \operatorname{Sech}\left[\frac{1}{2} \operatorname{ArcTanh}[a x]\right]^2+3 \operatorname{ArcTanh}[a x] \operatorname{Sech}\left[\frac{1}{2} \operatorname{ArcTanh}[a x]\right]^4-\frac{16\left(1-a^2 x^2\right)^{3/2} \operatorname{Sinh}\left[\frac{1}{2} \operatorname{ArcTanh}[a x]\right]^4}{a^3 x^3}+8 \operatorname{Tanh}\left[\frac{1}{2} \operatorname{ArcTanh}[a x]\right]\right)+$$

$$\frac{1}{5760} a^6 \left(-76 \operatorname{Coth}\left[\frac{1}{2} \operatorname{ArcTanh}[a x]\right]-90 \operatorname{ArcTanh}[a x] \operatorname{Csch}\left[\frac{1}{2} \operatorname{ArcTanh}[a x]\right]^2-\frac{26 a x \operatorname{Csch}\left[\frac{1}{2} \operatorname{ArcTanh}[a x]\right]^4}{\sqrt{1-a^2 x^2}}-90 \operatorname{ArcTanh}[a x] \operatorname{Csch}\left[\frac{1}{2} \operatorname{ArcTanh}[a x]\right]^4-\frac{3 a x \operatorname{Csch}\left[\frac{1}{2} \operatorname{ArcTanh}[a x]\right]^6}{\sqrt{1-a^2 x^2}}-15 \operatorname{ArcTanh}[a x] \operatorname{Csch}\left[\frac{1}{2} \operatorname{ArcTanh}[a x]\right]^6-360 \operatorname{ArcTanh}[a x] \operatorname{Log}\left[1-e^{-\operatorname{ArcTanh}[a x]}\right]+360 \operatorname{ArcTanh}[a x] \operatorname{Log}\left[1+e^{-\operatorname{ArcTanh}[a x]}\right]-360 \operatorname{PolyLog}\left[2,-e^{-\operatorname{ArcTanh}[a x]}\right]+360 \operatorname{PolyLog}\left[2, e^{-\operatorname{ArcTanh}[a x]}\right]-90 \operatorname{ArcTanh}[a x] \operatorname{Sech}\left[\frac{1}{2} \operatorname{ArcTanh}[a x]\right]^2+90 \operatorname{ArcTanh}[a x] \operatorname{Sech}\left[\frac{1}{2} \operatorname{ArcTanh}[a x]\right]^4-15 \operatorname{ArcTanh}[a x] \operatorname{Sech}\left[\frac{1}{2} \operatorname{ArcTanh}[a x]\right]^6-\frac{416\left(1-a^2 x^2\right)^{3/2} \operatorname{Sinh}\left[\frac{1}{2} \operatorname{ArcTanh}[a x]\right]^4}{a^3 x^3}+76 \operatorname{Tanh}\left[\frac{1}{2} \operatorname{ArcTanh}[a x]\right]+6 \operatorname{Sech}\left[\frac{1}{2} \operatorname{ArcTanh}[a x]\right]^4 \operatorname{Tanh}\left[\frac{1}{2} \operatorname{ArcTanh}[a x]\right]\right)$$

Problem 502: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{\operatorname{ArcTanh}[a x]}{c+d x^2} \, dx$$

Optimal (type 4, 429 leaves, 17 steps):

$$-\frac{\text{Log}[1-a x]\,\text{Log}\left[\frac{a\left(\sqrt{-c}-\sqrt{d}\,x\right)}{a\sqrt{-c}-\sqrt{d}}\right]}{4\sqrt{-c}\,\sqrt{d}}+\frac{\text{Log}[1+a x]\,\text{Log}\left[\frac{a\left(\sqrt{-c}-\sqrt{d}\,x\right)}{a\sqrt{-c}+\sqrt{d}}\right]}{4\sqrt{-c}\,\sqrt{d}}-$$

$$\frac{\text{Log}[1+a x]\,\text{Log}\left[\frac{a\left(\sqrt{-c}+\sqrt{d}\,x\right)}{a\sqrt{-c}-\sqrt{d}}\right]}{4\sqrt{-c}\,\sqrt{d}}+\frac{\text{Log}[1-a x]\,\text{Log}\left[\frac{a\left(\sqrt{-c}+\sqrt{d}\,x\right)}{a\sqrt{-c}+\sqrt{d}}\right]}{4\sqrt{-c}\,\sqrt{d}}-\frac{\text{PolyLog}\left[2,-\frac{\sqrt{d}\,(1-a x)}{a\sqrt{-c}-\sqrt{d}}\right]}{4\sqrt{-c}\,\sqrt{d}}+$$

$$\frac{\text{PolyLog}\left[2,\frac{\sqrt{d}\,(1-a x)}{a\sqrt{-c}+\sqrt{d}}\right]}{4\sqrt{-c}\,\sqrt{d}}-\frac{\text{PolyLog}\left[2,-\frac{\sqrt{d}\,(1+a x)}{a\sqrt{-c}-\sqrt{d}}\right]}{4\sqrt{-c}\,\sqrt{d}}+\frac{\text{PolyLog}\left[2,\frac{\sqrt{d}\,(1+a x)}{a\sqrt{-c}+\sqrt{d}}\right]}{4\sqrt{-c}\,\sqrt{d}}$$

Result (type 4, 662 leaves):

$$-\frac{1}{4\sqrt{a^2 c d}}a\left(-2\,i\,\text{ArcCos}\left[\frac{-a^2 c+d}{a^2 c+d}\right]\text{ArcTan}\left[\frac{a d x}{\sqrt{a^2 c d}}\right]+4\,\text{ArcTan}\left[\frac{a c}{\sqrt{a^2 c d}\,x}\right]\text{ArcTanh}[a x]-\right.$$

$$\left(\text{ArcCos}\left[\frac{-a^2 c+d}{a^2 c+d}\right]+2\,\text{ArcTan}\left[\frac{a d x}{\sqrt{a^2 c d}}\right]\right)\text{Log}\left[\frac{2\,i\,a c\left(i\,d+\sqrt{a^2 c d}\right)(-1+a x)}{\left(a^2 c+d\right)\left(a c+i\sqrt{a^2 c d}\,x\right)}\right]-$$

$$\left(\text{ArcCos}\left[\frac{-a^2 c+d}{a^2 c+d}\right]-2\,\text{ArcTan}\left[\frac{a d x}{\sqrt{a^2 c d}}\right]\right)\text{Log}\left[\frac{2 a c\left(d+i\sqrt{a^2 c d}\right)(1+a x)}{\left(a^2 c+d\right)\left(a c+i\sqrt{a^2 c d}\,x\right)}\right]+$$

$$\left(\text{ArcCos}\left[\frac{-a^2 c+d}{a^2 c+d}\right]+2\left(\text{ArcTan}\left[\frac{a c}{\sqrt{a^2 c d}\,x}\right]+\text{ArcTan}\left[\frac{a d x}{\sqrt{a^2 c d}}\right]\right)\right)$$

$$\text{Log}\left[\frac{\sqrt{2}\,\sqrt{a^2 c d}\,e^{-\text{ArcTanh}[a x]}}{\sqrt{a^2 c+d}\,\sqrt{a^2 c-d+\left(a^2 c+d\right)\text{Cosh}[2\,\text{ArcTanh}[a x]]}}\right]+$$

$$\left(\text{ArcCos}\left[\frac{-a^2 c+d}{a^2 c+d}\right]-2\left(\text{ArcTan}\left[\frac{a c}{\sqrt{a^2 c d}\,x}\right]+\text{ArcTan}\left[\frac{a d x}{\sqrt{a^2 c d}}\right]\right)\right)$$

$$\text{Log}\left[\frac{\sqrt{2}\,\sqrt{a^2 c d}\,e^{\text{ArcTanh}[a x]}}{\sqrt{a^2 c+d}\,\sqrt{a^2 c-d+\left(a^2 c+d\right)\text{Cosh}[2\,\text{ArcTanh}[a x]]}}\right]+$$

$$i\left(-\text{PolyLog}\left[2,\frac{\left(-a^2 c+d-2\,i\sqrt{a^2 c d}\right)\left(i\,a c+\sqrt{a^2 c d}\,x\right)}{\left(a^2 c+d\right)\left(-i\,a c+\sqrt{a^2 c d}\,x\right)}\right]+\right.$$

$$\left.\left.\text{PolyLog}\left[2,\frac{\left(-a^2 c+d+2\,i\sqrt{a^2 c d}\right)\left(i\,a c+\sqrt{a^2 c d}\,x\right)}{\left(a^2 c+d\right)\left(-i\,a c+\sqrt{a^2 c d}\,x\right)}\right]\right)\right)$$

Problem 504: Result more than twice size of optimal antiderivative.

$$\int\frac{\text{ArcTanh}[a x]}{\left(c+d x^2\right)^3}\,dx$$

Optimal (type 4, 657 leaves, 23 steps):

$$\frac{a}{8 c \left(a^2 c+d\right) \left(c+d x^2\right)}+\frac{x \operatorname{ArcTanh}[a x]}{4 c \left(c+d x^2\right)^2}+\frac{3 x \operatorname{ArcTanh}[a x]}{8 c^2 \left(c+d x^2\right)}+\frac{3 \operatorname{ArcTan}\left[\frac{\sqrt{d} x}{\sqrt{c}}\right] \operatorname{ArcTanh}[a x]}{8 c^{5/2} \sqrt{d}}+$$

$$\frac{3 i \operatorname{Log}\left[\frac{\sqrt{d} (1-a x)}{i a \sqrt{c}+\sqrt{d}}\right] \operatorname{Log}\left[1-\frac{i \sqrt{d} x}{\sqrt{c}}\right]}{32 c^{5/2} \sqrt{d}}-\frac{3 i \operatorname{Log}\left[-\frac{\sqrt{d} (1+a x)}{i a \sqrt{c}-\sqrt{d}}\right] \operatorname{Log}\left[1-\frac{i \sqrt{d} x}{\sqrt{c}}\right]}{32 c^{5/2} \sqrt{d}}-$$

$$\frac{3 i \operatorname{Log}\left[-\frac{\sqrt{d} (1-a x)}{i a \sqrt{c}-\sqrt{d}}\right] \operatorname{Log}\left[1+\frac{i \sqrt{d} x}{\sqrt{c}}\right]}{32 c^{5/2} \sqrt{d}}+\frac{3 i \operatorname{Log}\left[\frac{\sqrt{d} (1+a x)}{i a \sqrt{c}+\sqrt{d}}\right] \operatorname{Log}\left[1+\frac{i \sqrt{d} x}{\sqrt{c}}\right]}{32 c^{5/2} \sqrt{d}}+$$

$$\frac{a \left(5 a^2 c+3 d\right) \operatorname{Log}\left[1-a^2 x^2\right]}{16 c^2 \left(a^2 c+d\right)^2}-\frac{a \left(5 a^2 c+3 d\right) \operatorname{Log}\left[c+d x^2\right]}{16 c^2 \left(a^2 c+d\right)^2}+\frac{3 i \operatorname{PolyLog}\left[2,\frac{a \left(\sqrt{c}-i \sqrt{d} x\right)}{a \sqrt{c}-i \sqrt{d}}\right]}{32 c^{5/2} \sqrt{d}}-$$

$$\frac{3 i \operatorname{PolyLog}\left[2,\frac{a \left(\sqrt{c}-i \sqrt{d} x\right)}{a \sqrt{c}+i \sqrt{d}}\right]}{32 c^{5/2} \sqrt{d}}+\frac{3 i \operatorname{PolyLog}\left[2,\frac{a \left(\sqrt{c}+i \sqrt{d} x\right)}{a \sqrt{c}-i \sqrt{d}}\right]}{32 c^{5/2} \sqrt{d}}-\frac{3 i \operatorname{PolyLog}\left[2,\frac{a \left(\sqrt{c}+i \sqrt{d} x\right)}{a \sqrt{c}+i \sqrt{d}}\right]}{32 c^{5/2} \sqrt{d}}$$

Result (type 4, 1840 leaves):

$$a^5 \left(-\frac{5 \operatorname{Log}\left[1+\frac{\left(a^2 c+d\right) \operatorname{Cosh}[2 \operatorname{ArcTanh}[a x]]}{a^2 c-d}\right]}{16 a^2 c \left(a^2 c+d\right)^2}-\frac{3 d \operatorname{Log}\left[1+\frac{\left(a^2 c+d\right) \operatorname{Cosh}[2 \operatorname{ArcTanh}[a x]]}{a^2 c-d}\right]}{16 a^4 c^2 \left(a^2 c+d\right)^2}-\right.$$

$$\frac{1}{32 a^2 c \sqrt{a^2 c d} \left(a^2 c+d\right)} 3 \left(-2 i \operatorname{ArcCos}\left[-\frac{a^2 c-d}{a^2 c+d}\right] \operatorname{ArcTan}\left[\frac{a d x}{\sqrt{a^2 c d}}\right]+\right.$$

$$4 \operatorname{ArcTan}\left[\frac{a c}{\sqrt{a^2 c d} x}\right] \operatorname{ArcTanh}[a x]-\left(\operatorname{ArcCos}\left[-\frac{a^2 c-d}{a^2 c+d}\right]-2 \operatorname{ArcTan}\left[\frac{a d x}{\sqrt{a^2 c d}}\right]\right)$$

$$\operatorname{Log}\left[1-\frac{\left(a^2 c-d-2 i \sqrt{a^2 c d}\right) \left(2 a^2 c-2 i a \sqrt{a^2 c d} x\right)}{\left(a^2 c+d\right) \left(2 a^2 c+2 i a \sqrt{a^2 c d} x\right)}\right]+\left(-\operatorname{ArcCos}\left[-\frac{a^2 c-d}{a^2 c+d}\right]-\right.$$

$$\left.2 \operatorname{ArcTan}\left[\frac{a d x}{\sqrt{a^2 c d}}\right]\right) \operatorname{Log}\left[1-\frac{\left(a^2 c-d+2 i \sqrt{a^2 c d}\right) \left(2 a^2 c-2 i a \sqrt{a^2 c d} x\right)}{\left(a^2 c+d\right) \left(2 a^2 c+2 i a \sqrt{a^2 c d} x\right)}\right]+$$

$$\left(\operatorname{ArcCos}\left[-\frac{a^2 c-d}{a^2 c+d}\right]+2 i \left(-i \operatorname{ArcTan}\left[\frac{a c}{\sqrt{a^2 c d} x}\right]-i \operatorname{ArcTan}\left[\frac{a d x}{\sqrt{a^2 c d}}\right]\right)\right)$$

$$\operatorname{Log}\left[\frac{\sqrt{2} \sqrt{a^2 c d} e^{-\operatorname{ArcTanh}[a x]}}{\sqrt{a^2 c+d} \sqrt{a^2 c-d+\left(a^2 c+d\right) \operatorname{Cosh}[2 \operatorname{ArcTanh}[a x]]}}\right]+$$

$$\left(\operatorname{ArcCos}\left[-\frac{a^2 c-d}{a^2 c+d}\right]-2 i \left(-i \operatorname{ArcTan}\left[\frac{a c}{\sqrt{a^2 c d} x}\right]-i \operatorname{ArcTan}\left[\frac{a d x}{\sqrt{a^2 c d}}\right]\right)\right)$$

$$\operatorname{Log}\left[\frac{\sqrt{2} \sqrt{a^2 c d} e^{\operatorname{ArcTanh}[a x]}}{\sqrt{a^2 c+d} \sqrt{a^2 c-d+\left(a^2 c+d\right) \operatorname{Cosh}[2 \operatorname{ArcTanh}[a x]]}}\right]+$$

$$\mathrm{i}\left(\text{PolyLog}\left[2,\frac{\left(a^2 c-d-2\,\mathrm{i}\sqrt{a^2 c d}\right)\left(2 a^2 c-2\,\mathrm{i}\,a\sqrt{a^2 c d}\,x\right)}{\left(a^2 c+d\right)\left(2 a^2 c+2\,\mathrm{i}\,a\sqrt{a^2 c d}\,x\right)}\right]-\right.$$

$$\left.\left.\text{PolyLog}\left[2,\frac{\left(a^2 c-d+2\,\mathrm{i}\sqrt{a^2 c d}\right)\left(2 a^2 c-2\,\mathrm{i}\,a\sqrt{a^2 c d}\,x\right)}{\left(a^2 c+d\right)\left(2 a^2 c+2\,\mathrm{i}\,a\sqrt{a^2 c d}\,x\right)}\right]\right)\right)-$$

$$\frac{1}{32 a^4 c^2\sqrt{a^2 c d}\left(a^2 c+d\right)}3 d\left(-2\,\mathrm{i}\,\text{ArcCos}\left[-\frac{a^2 c-d}{a^2 c+d}\right]\text{ArcTan}\left[\frac{a d x}{\sqrt{a^2 c d}}\right]+\right.$$

$$4\,\text{ArcTan}\left[\frac{a c}{\sqrt{a^2 c d}\,x}\right]\text{ArcTanh}[a x]-\left(\text{ArcCos}\left[-\frac{a^2 c-d}{a^2 c+d}\right]-2\,\text{ArcTan}\left[\frac{a d x}{\sqrt{a^2 c d}}\right]\right)$$

$$\text{Log}\left[1-\frac{\left(a^2 c-d-2\,\mathrm{i}\sqrt{a^2 c d}\right)\left(2 a^2 c-2\,\mathrm{i}\,a\sqrt{a^2 c d}\,x\right)}{\left(a^2 c+d\right)\left(2 a^2 c+2\,\mathrm{i}\,a\sqrt{a^2 c d}\,x\right)}\right]+$$

$$\left(-\text{ArcCos}\left[-\frac{a^2 c-d}{a^2 c+d}\right]-2\,\text{ArcTan}\left[\frac{a d x}{\sqrt{a^2 c d}}\right]\right)$$

$$\text{Log}\left[1-\frac{\left(a^2 c-d+2\,\mathrm{i}\sqrt{a^2 c d}\right)\left(2 a^2 c-2\,\mathrm{i}\,a\sqrt{a^2 c d}\,x\right)}{\left(a^2 c+d\right)\left(2 a^2 c+2\,\mathrm{i}\,a\sqrt{a^2 c d}\,x\right)}\right]+$$

$$\left(\text{ArcCos}\left[-\frac{a^2 c-d}{a^2 c+d}\right]+2\,\mathrm{i}\left(-\mathrm{i}\,\text{ArcTan}\left[\frac{a c}{\sqrt{a^2 c d}\,x}\right]-\mathrm{i}\,\text{ArcTan}\left[\frac{a d x}{\sqrt{a^2 c d}}\right]\right)\right)$$

$$\text{Log}\left[\frac{\sqrt{2}\sqrt{a^2 c d}\,e^{-\text{ArcTanh}[a x]}}{\sqrt{a^2 c+d}\sqrt{a^2 c-d+\left(a^2 c+d\right)\text{Cosh}[2\,\text{ArcTanh}[a x]]}}\right]+$$

$$\left(\text{ArcCos}\left[-\frac{a^2 c-d}{a^2 c+d}\right]-2\,\mathrm{i}\left(-\mathrm{i}\,\text{ArcTan}\left[\frac{a c}{\sqrt{a^2 c d}\,x}\right]-\mathrm{i}\,\text{ArcTan}\left[\frac{a d x}{\sqrt{a^2 c d}}\right]\right)\right)$$

$$\text{Log}\left[\frac{\sqrt{2}\sqrt{a^2 c d}\,e^{\text{ArcTanh}[a x]}}{\sqrt{a^2 c+d}\sqrt{a^2 c-d+\left(a^2 c+d\right)\text{Cosh}[2\,\text{ArcTanh}[a x]]}}\right]+$$

$$\mathrm{i}\left(\text{PolyLog}\left[2,\frac{\left(a^2 c-d-2\,\mathrm{i}\sqrt{a^2 c d}\right)\left(2 a^2 c-2\,\mathrm{i}\,a\sqrt{a^2 c d}\,x\right)}{\left(a^2 c+d\right)\left(2 a^2 c+2\,\mathrm{i}\,a\sqrt{a^2 c d}\,x\right)}\right]-\right.$$

$$\left.\left.\text{PolyLog}\left[2,\frac{\left(a^2 c-d+2\,\mathrm{i}\sqrt{a^2 c d}\right)\left(2 a^2 c-2\,\mathrm{i}\,a\sqrt{a^2 c d}\,x\right)}{\left(a^2 c+d\right)\left(2 a^2 c+2\,\mathrm{i}\,a\sqrt{a^2 c d}\,x\right)}\right]\right)\right)+$$

$$\left(d\,\text{ArcTanh}[a x]\,\text{Sinh}[2\,\text{ArcTanh}[a x]]\right)\Big/\left(2 a^2 c\left(a^2 c+d\right)\right.$$

$$\left.\left(a^2 c-d+a^2 c\,\text{Cosh}[2\,\text{ArcTanh}[a x]]+d\,\text{Cosh}[2\,\text{ArcTanh}[a x]]\right)^2\right)+$$

$$\left(2 a^2 c d+5 a^4 c^2\,\text{ArcTanh}[a x]\,\text{Sinh}[2\,\text{ArcTanh}[a x]]+\right.$$

$$\left.8 a^2 c d\,\text{ArcTanh}[a x]\,\text{Sinh}[2\,\text{ArcTanh}[a x]]+3 d^2\,\text{ArcTanh}[a x]\,\text{Sinh}[2\,\text{ArcTanh}[a x]]\right)\Big/$$

$$\left.\left(8 a^4 c^2\left(a^2 c+d\right)^2\left(a^2 c-d+a^2 c\,\text{Cosh}[2\,\text{ArcTanh}[a x]]+d\,\text{Cosh}[2\,\text{ArcTanh}[a x]]\right)\right)\right)$$

Problem 506: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.

$$\int \frac{\text{ArcTanh}[b\,x]}{1-x^2}\,dx$$

Optimal (type 4, 171 leaves, 17 steps):

$$\frac{1}{4}\text{Log}\left[-\frac{b\,(1-x)}{1-b}\right]\text{Log}[1-b\,x]-\frac{1}{4}\text{Log}\left[\frac{b\,(1+x)}{1+b}\right]\text{Log}[1-b\,x]-\frac{1}{4}\text{Log}\left[\frac{b\,(1-x)}{1+b}\right]\text{Log}[1+b\,x]+\frac{1}{4}\text{Log}\left[-\frac{b\,(1+x)}{1-b}\right]\text{Log}[1+b\,x]+\frac{1}{4}\text{PolyLog}\left[2,\frac{1-b\,x}{1-b}\right]-\frac{1}{4}\text{PolyLog}\left[2,\frac{1-b\,x}{1+b}\right]+\frac{1}{4}\text{PolyLog}\left[2,\frac{1+b\,x}{1-b}\right]-\frac{1}{4}\text{PolyLog}\left[2,\frac{1+b\,x}{1+b}\right]$$

Result (type 4, 576 leaves):

$$-\frac{1}{4\sqrt{-b^2}}b\left(2\,i\,\text{ArcCos}\left[\frac{1+b^2}{1-b^2}\right]\text{ArcTan}\left[\frac{b\,x}{\sqrt{-b^2}}\right]-4\,\text{ArcTan}\left[\frac{\sqrt{-b^2}}{b\,x}\right]\text{ArcTanh}[b\,x]-\left(\text{ArcCos}\left[\frac{1+b^2}{1-b^2}\right]-2\,\text{ArcTan}\left[\frac{b\,x}{\sqrt{-b^2}}\right]\right)\text{Log}\left[\frac{2\,b\left(-i+\sqrt{-b^2}\right)(-1+b\,x)}{(-1+b^2)\left(-i\,b+\sqrt{-b^2}\,x\right)}\right]-\left(\text{ArcCos}\left[\frac{1+b^2}{1-b^2}\right]+2\,\text{ArcTan}\left[\frac{b\,x}{\sqrt{-b^2}}\right]\right)\text{Log}\left[\frac{2\,b\left(i+\sqrt{-b^2}\right)(1+b\,x)}{(-1+b^2)\left(-i\,b+\sqrt{-b^2}\,x\right)}\right]+\left(\text{ArcCos}\left[\frac{1+b^2}{1-b^2}\right]-2\left(\text{ArcTan}\left[\frac{\sqrt{-b^2}}{b\,x}\right]+\text{ArcTan}\left[\frac{b\,x}{\sqrt{-b^2}}\right]\right)\right)\text{Log}\left[\frac{\sqrt{2}\,\sqrt{-b^2}\,e^{-\text{ArcTanh}[b\,x]}}{\sqrt{-1+b^2}\,\sqrt{1+b^2+(-1+b^2)\,\text{Cosh}[2\,\text{ArcTanh}[b\,x]]}}\right]+\left(\text{ArcCos}\left[\frac{1+b^2}{1-b^2}\right]+2\left(\text{ArcTan}\left[\frac{\sqrt{-b^2}}{b\,x}\right]+\text{ArcTan}\left[\frac{b\,x}{\sqrt{-b^2}}\right]\right)\right)\text{Log}\left[\frac{\sqrt{2}\,\sqrt{-b^2}\,e^{\text{ArcTanh}[b\,x]}}{\sqrt{-1+b^2}\,\sqrt{1+b^2+(-1+b^2)\,\text{Cosh}[2\,\text{ArcTanh}[b\,x]]}}\right]+i\left(\text{PolyLog}\left[2,\frac{\left(1+b^2-2\,i\,\sqrt{-b^2}\right)\left(b-i\,\sqrt{-b^2}\,x\right)}{(-1+b^2)\left(b+i\,\sqrt{-b^2}\,x\right)}\right]-\text{PolyLog}\left[2,\frac{\left(1+b^2+2\,i\,\sqrt{-b^2}\right)\left(b-i\,\sqrt{-b^2}\,x\right)}{(-1+b^2)\left(b+i\,\sqrt{-b^2}\,x\right)}\right]\right)\right)$$

Problem 507: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.

$$\int \frac{\text{ArcTanh}[a + b\,x]}{1 - x^2}\,dx$$

Optimal (type 4, 203 leaves, 17 steps):

$$\frac{1}{4}\,\text{Log}\left[-\frac{b\,(1-x)}{1-a-b}\right]\,\text{Log}[1-a-b\,x] - \frac{1}{4}\,\text{Log}\left[\frac{b\,(1+x)}{1-a+b}\right]\,\text{Log}[1-a-b\,x] - \frac{1}{4}\,\text{Log}\left[\frac{b\,(1-x)}{1+a+b}\right]\,\text{Log}[1+a+b\,x] + \frac{1}{4}\,\text{Log}\left[-\frac{b\,(1+x)}{1+a-b}\right]\,\text{Log}[1+a+b\,x] + \frac{1}{4}\,\text{PolyLog}\left[2,\,\frac{1-a-b\,x}{1-a-b}\right] - \frac{1}{4}\,\text{PolyLog}\left[2,\,\frac{1-a-b\,x}{1-a+b}\right] + \frac{1}{4}\,\text{PolyLog}\left[2,\,\frac{1+a+b\,x}{1+a-b}\right] - \frac{1}{4}\,\text{PolyLog}\left[2,\,\frac{1+a+b\,x}{1+a+b}\right]$$

Result (type 4, 646 leaves):

$$-\frac{1}{4\left(-1+a^2\right)}\left(-2\,\text{ArcTanh}\left[\frac{-1+a}{b}\right]\text{ArcTanh}[x]+2\,a^2\,\text{ArcTanh}\left[\frac{-1+a}{b}\right]\text{ArcTanh}[x]+\right.$$
$$2\,\text{ArcTanh}\left[\frac{1+a}{b}\right]\text{ArcTanh}[x]-2\,a^2\,\text{ArcTanh}\left[\frac{1+a}{b}\right]\text{ArcTanh}[x]-2\,b\,\text{ArcTanh}[x]^2+$$
$$b\sqrt{\frac{-1+2\,a-a^2+b^2}{b^2}}\,e^{\text{ArcTanh}\left[\frac{-1+a}{b}\right]}\,\text{ArcTanh}[x]^2+a\,b\sqrt{\frac{-1+2\,a-a^2+b^2}{b^2}}$$
$$e^{\text{ArcTanh}\left[\frac{-1+a}{b}\right]}\,\text{ArcTanh}[x]^2+b\sqrt{-\frac{1+2\,a+a^2-b^2}{b^2}}\,e^{\text{ArcTanh}\left[\frac{1+a}{b}\right]}\,\text{ArcTanh}[x]^2-$$
$$a\,b\sqrt{-\frac{1+2\,a+a^2-b^2}{b^2}}\,e^{\text{ArcTanh}\left[\frac{1+a}{b}\right]}\,\text{ArcTanh}[x]^2+4\,\text{ArcTanh}[x]\,\text{ArcTanh}[a+b\,x]-$$
$$4\,a^2\,\text{ArcTanh}[x]\,\text{ArcTanh}[a+b\,x]+2\,\text{ArcTanh}\left[\frac{-1+a}{b}\right]\text{Log}\left[1-e^{2\left(\text{ArcTanh}\left[\frac{-1+a}{b}\right]+\text{ArcTanh}[x]\right)}\right]-$$
$$2\,a^2\,\text{ArcTanh}\left[\frac{-1+a}{b}\right]\text{Log}\left[1-e^{2\left(\text{ArcTanh}\left[\frac{-1+a}{b}\right]+\text{ArcTanh}[x]\right)}\right]+$$
$$2\,\text{ArcTanh}[x]\,\text{Log}\left[1-e^{2\left(\text{ArcTanh}\left[\frac{-1+a}{b}\right]+\text{ArcTanh}[x]\right)}\right]-2\,a^2\,\text{ArcTanh}[x]$$
$$\text{Log}\left[1-e^{2\left(\text{ArcTanh}\left[\frac{-1+a}{b}\right]+\text{ArcTanh}[x]\right)}\right]-2\,\text{ArcTanh}\left[\frac{1+a}{b}\right]\text{Log}\left[1-e^{2\left(\text{ArcTanh}\left[\frac{1+a}{b}\right]+\text{ArcTanh}[x]\right)}\right]+$$
$$2\,a^2\,\text{ArcTanh}\left[\frac{1+a}{b}\right]\text{Log}\left[1-e^{2\left(\text{ArcTanh}\left[\frac{1+a}{b}\right]+\text{ArcTanh}[x]\right)}\right]-2\,\text{ArcTanh}[x]$$
$$\text{Log}\left[1-e^{2\left(\text{ArcTanh}\left[\frac{1+a}{b}\right]+\text{ArcTanh}[x]\right)}\right]+2\,a^2\,\text{ArcTanh}[x]\,\text{Log}\left[1-e^{2\left(\text{ArcTanh}\left[\frac{1+a}{b}\right]+\text{ArcTanh}[x]\right)}\right]-$$
$$2\,\text{ArcTanh}\left[\frac{-1+a}{b}\right]\text{Log}\left[i\,\text{Sinh}\left[\text{ArcTanh}\left[\frac{1-a}{b}\right]-\text{ArcTanh}[x]\right]\right]+$$
$$2\,a^2\,\text{ArcTanh}\left[\frac{-1+a}{b}\right]\text{Log}\left[i\,\text{Sinh}\left[\text{ArcTanh}\left[\frac{1-a}{b}\right]-\text{ArcTanh}[x]\right]\right]+$$
$$2\,\text{ArcTanh}\left[\frac{1+a}{b}\right]\text{Log}\left[-i\,\text{Sinh}\left[\text{ArcTanh}\left[\frac{1+a}{b}\right]+\text{ArcTanh}[x]\right]\right]-$$
$$2\,a^2\,\text{ArcTanh}\left[\frac{1+a}{b}\right]\text{Log}\left[-i\,\text{Sinh}\left[\text{ArcTanh}\left[\frac{1+a}{b}\right]+\text{ArcTanh}[x]\right]\right]-$$
$$\left.\left(-1+a^2\right)\text{PolyLog}\left[2,\,e^{2\left(\text{ArcTanh}\left[\frac{-1+a}{b}\right]+\text{ArcTanh}[x]\right)}\right]+\left(-1+a^2\right)\text{PolyLog}\left[2,\,e^{2\left(\text{ArcTanh}\left[\frac{1+a}{b}\right]+\text{ArcTanh}[x]\right)}\right]\right)$$

Problem 508: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.

$$\int\frac{\text{ArcTanh}[x]}{a+b\,x}\,dx$$

Optimal (type 4, 86 leaves, 4 steps):

$$-\frac{\text{ArcTanh}[x]\,\text{Log}\left[\frac{2}{1+x}\right]}{b}+\frac{\text{ArcTanh}[x]\,\text{Log}\left[\frac{2\,(a+b\,x)}{(a+b)\,(1+x)}\right]}{b}+\frac{\text{PolyLog}\left[2,\,1-\frac{2}{1+x}\right]}{2\,b}-\frac{\text{PolyLog}\left[2,\,1-\frac{2\,(a+b\,x)}{(a+b)\,(1+x)}\right]}{2\,b}$$

Result (type 4, 260 leaves):

$$\frac{1}{8\,b}\Bigg(-\pi^2+4\,\text{ArcTanh}\left[\frac{a}{b}\right]^2+4\,i\,\pi\,\text{ArcTanh}[x]+8\,\text{ArcTanh}\left[\frac{a}{b}\right]\text{ArcTanh}[x]+8\,\text{ArcTanh}[x]^2-4\,i\,\pi\,\text{Log}\left[1+e^{2\,\text{ArcTanh}[x]}\right]-8\,\text{ArcTanh}[x]\,\text{Log}\left[1+e^{2\,\text{ArcTanh}[x]}\right]+8\,\text{ArcTanh}\left[\frac{a}{b}\right]\text{Log}\left[1-e^{-2\left(\text{ArcTanh}\left[\frac{a}{b}\right]+\text{ArcTanh}[x]\right)}\right]+8\,\text{ArcTanh}[x]\,\text{Log}\left[1-e^{-2\left(\text{ArcTanh}\left[\frac{a}{b}\right]+\text{ArcTanh}[x]\right)}\right]+4\,i\,\pi\,\text{Log}\left[\frac{2}{\sqrt{1-x^2}}\right]+8\,\text{ArcTanh}[x]\,\text{Log}\left[\frac{2}{\sqrt{1-x^2}}\right]+4\,\text{ArcTanh}[x]\,\text{Log}\left[1-x^2\right]+8\,\text{ArcTanh}[x]\,\text{Log}\left[i\,\text{Sinh}\left[\text{ArcTanh}\left[\frac{a}{b}\right]+\text{ArcTanh}[x]\right]\right]-8\,\text{ArcTanh}\left[\frac{a}{b}\right]\text{Log}\left[2\,i\,\text{Sinh}\left[\text{ArcTanh}\left[\frac{a}{b}\right]+\text{ArcTanh}[x]\right]\right]-8\,\text{ArcTanh}[x]\,\text{Log}\left[2\,i\,\text{Sinh}\left[\text{ArcTanh}\left[\frac{a}{b}\right]+\text{ArcTanh}[x]\right]\right]-4\,\text{PolyLog}\left[2,\,-e^{2\,\text{ArcTanh}[x]}\right]-4\,\text{PolyLog}\left[2,\,e^{-2\left(\text{ArcTanh}\left[\frac{a}{b}\right]+\text{ArcTanh}[x]\right)}\right]\Bigg)$$

Problem 509: Result unnecessarily involves imaginary or complex numbers.

$$\int\frac{\text{ArcTanh}[x]}{a+b\,x^2}\,dx$$

Optimal (type 4, 397 leaves, 17 steps):

$$-\frac{\text{Log}[1-x]\,\text{Log}\left[\frac{\sqrt{-a}-\sqrt{b}\,x}{\sqrt{-a}-\sqrt{b}}\right]}{4\,\sqrt{-a}\,\sqrt{b}}+\frac{\text{Log}[1+x]\,\text{Log}\left[\frac{\sqrt{-a}-\sqrt{b}\,x}{\sqrt{-a}+\sqrt{b}}\right]}{4\,\sqrt{-a}\,\sqrt{b}}-\frac{\text{Log}[1+x]\,\text{Log}\left[\frac{\sqrt{-a}+\sqrt{b}\,x}{\sqrt{-a}-\sqrt{b}}\right]}{4\,\sqrt{-a}\,\sqrt{b}}+\frac{\text{Log}[1-x]\,\text{Log}\left[\frac{\sqrt{-a}+\sqrt{b}\,x}{\sqrt{-a}+\sqrt{b}}\right]}{4\,\sqrt{-a}\,\sqrt{b}}-\frac{\text{PolyLog}\left[2,\,-\frac{\sqrt{b}\,(1-x)}{\sqrt{-a}-\sqrt{b}}\right]}{4\,\sqrt{-a}\,\sqrt{b}}+\frac{\text{PolyLog}\left[2,\,\frac{\sqrt{b}\,(1-x)}{\sqrt{-a}+\sqrt{b}}\right]}{4\,\sqrt{-a}\,\sqrt{b}}-\frac{\text{PolyLog}\left[2,\,-\frac{\sqrt{b}\,(1+x)}{\sqrt{-a}-\sqrt{b}}\right]}{4\,\sqrt{-a}\,\sqrt{b}}+\frac{\text{PolyLog}\left[2,\,\frac{\sqrt{b}\,(1+x)}{\sqrt{-a}+\sqrt{b}}\right]}{4\,\sqrt{-a}\,\sqrt{b}}$$

Result (type 4, 485 leaves):

$$-\frac{1}{4\sqrt{a\,b}}\left(-2\,\mathrm{i}\,\mathrm{ArcCos}\left[\frac{-a+b}{a+b}\right]\mathrm{ArcTan}\left[\frac{b\,x}{\sqrt{a\,b}}\right]+4\,\mathrm{ArcTan}\left[\frac{a}{\sqrt{a\,b}\,x}\right]\mathrm{ArcTanh}[x]-\right.$$
$$\left(\mathrm{ArcCos}\left[\frac{-a+b}{a+b}\right]+2\,\mathrm{ArcTan}\left[\frac{b\,x}{\sqrt{a\,b}}\right]\right)\mathrm{Log}\left[\frac{2\,\mathrm{i}\,a\left(\mathrm{i}\,b+\sqrt{a\,b}\right)(-1+x)}{(a+b)\left(a+\mathrm{i}\sqrt{a\,b}\,x\right)}\right]-$$
$$\left(\mathrm{ArcCos}\left[\frac{-a+b}{a+b}\right]-2\,\mathrm{ArcTan}\left[\frac{b\,x}{\sqrt{a\,b}}\right]\right)\mathrm{Log}\left[\frac{2\,a\left(b+\mathrm{i}\sqrt{a\,b}\right)(1+x)}{(a+b)\left(a+\mathrm{i}\sqrt{a\,b}\,x\right)}\right]+$$
$$\left(\mathrm{ArcCos}\left[\frac{-a+b}{a+b}\right]+2\left(\mathrm{ArcTan}\left[\frac{a}{\sqrt{a\,b}\,x}\right]+\mathrm{ArcTan}\left[\frac{b\,x}{\sqrt{a\,b}}\right]\right)\right)$$
$$\mathrm{Log}\left[\frac{\sqrt{2}\,\sqrt{a\,b}\,e^{-\mathrm{ArcTanh}[x]}}{\sqrt{a+b}\,\sqrt{a-b+(a+b)\,\mathrm{Cosh}[2\,\mathrm{ArcTanh}[x]]}}\right]+$$
$$\left(\mathrm{ArcCos}\left[\frac{-a+b}{a+b}\right]-2\left(\mathrm{ArcTan}\left[\frac{a}{\sqrt{a\,b}\,x}\right]+\mathrm{ArcTan}\left[\frac{b\,x}{\sqrt{a\,b}}\right]\right)\right)$$
$$\mathrm{Log}\left[\frac{\sqrt{2}\,\sqrt{a\,b}\,e^{\mathrm{ArcTanh}[x]}}{\sqrt{a+b}\,\sqrt{a-b+(a+b)\,\mathrm{Cosh}[2\,\mathrm{ArcTanh}[x]]}}\right]+$$
$$\mathrm{i}\left(-\mathrm{PolyLog}\left[2,\frac{\left(-a+b-2\,\mathrm{i}\sqrt{a\,b}\right)\left(\mathrm{i}\,a+\sqrt{a\,b}\,x\right)}{(a+b)\left(-\mathrm{i}\,a+\sqrt{a\,b}\,x\right)}\right]+\right.$$
$$\left.\left.\mathrm{PolyLog}\left[2,\frac{\left(-a+b+2\,\mathrm{i}\sqrt{a\,b}\right)\left(\mathrm{i}\,a+\sqrt{a\,b}\,x\right)}{(a+b)\left(-\mathrm{i}\,a+\sqrt{a\,b}\,x\right)}\right]\right)\right)$$

Problem 510: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.

$$\int\frac{\mathrm{ArcTanh}[x]}{a+b\,x+c\,x^2}\,dx$$

Optimal (type 4, 258 leaves, 10 steps):

$$\frac{\mathrm{ArcTanh}[x]\,\mathrm{Log}\left[\frac{2\left(b-\sqrt{b^2-4\,a\,c}+2\,c\,x\right)}{\left(b+2\,c-\sqrt{b^2-4\,a\,c}\right)(1+x)}\right]}{\sqrt{b^2-4\,a\,c}}-\frac{\mathrm{ArcTanh}[x]\,\mathrm{Log}\left[\frac{2\left(b+\sqrt{b^2-4\,a\,c}+2\,c\,x\right)}{\left(b+2\,c+\sqrt{b^2-4\,a\,c}\right)(1+x)}\right]}{\sqrt{b^2-4\,a\,c}}-$$
$$\frac{\mathrm{PolyLog}\left[2,1-\frac{2\left(b-\sqrt{b^2-4\,a\,c}+2\,c\,x\right)}{\left(b+2\,c-\sqrt{b^2-4\,a\,c}\right)(1+x)}\right]}{2\sqrt{b^2-4\,a\,c}}+\frac{\mathrm{PolyLog}\left[2,1-\frac{2\left(b+\sqrt{b^2-4\,a\,c}+2\,c\,x\right)}{\left(b+2\,c+\sqrt{b^2-4\,a\,c}\right)(1+x)}\right]}{2\sqrt{b^2-4\,a\,c}}$$

Result (type 4, 910 leaves):

$$\frac{1}{2\sqrt{-b^2+4ac}\,\left(b^2-4c^2\right)}\Bigg(2\Bigg(\sqrt{-b^2+4ac}\,\Bigg(b\Bigg(\sqrt{\frac{c\,(a+b+c)}{-b^2+4ac}}\;e^{i\,\text{ArcTan}\left[\frac{-b-2c}{\sqrt{-b^2+4ac}}\right]}-\sqrt{\frac{c\,(a-b+c)}{-b^2+4ac}}\;e^{i\,\text{ArcTan}\left[\frac{-b+2c}{\sqrt{-b^2+4ac}}\right]}\Bigg)-$$
$$2c\Bigg(-1+\sqrt{\frac{c\,(a+b+c)}{-b^2+4ac}}\;e^{i\,\text{ArcTan}\left[\frac{-b-2c}{\sqrt{-b^2+4ac}}\right]}+\sqrt{\frac{c\,(a-b+c)}{-b^2+4ac}}\;e^{i\,\text{ArcTan}\left[\frac{-b+2c}{\sqrt{-b^2+4ac}}\right]}\Bigg)\Bigg)$$
$$\text{ArcTan}\Big[\frac{b+2cx}{\sqrt{-b^2+4ac}}\Big]^2+\left(b^2-4c^2\right)\text{ArcTan}\Big[\frac{b+2cx}{\sqrt{-b^2+4ac}}\Big]$$
$$\Bigg(-i\,\text{ArcTan}\Big[\frac{-b-2c}{\sqrt{-b^2+4ac}}\Big]+i\,\text{ArcTan}\Big[\frac{-b+2c}{\sqrt{-b^2+4ac}}\Big]+2\,\text{ArcTanh}[x]+$$
$$\text{Log}\Big[1-e^{2i\left(\text{ArcTan}\left[\frac{-b-2c}{\sqrt{-b^2+4ac}}\right]+\text{ArcTan}\left[\frac{b+2cx}{\sqrt{-b^2+4ac}}\right]\right)}\Big]-\text{Log}\Big[1-e^{2i\left(\text{ArcTan}\left[\frac{-b+2c}{\sqrt{-b^2+4ac}}\right]+\text{ArcTan}\left[\frac{b+2cx}{\sqrt{-b^2+4ac}}\right]\right)}\Big]\Bigg)+$$
$$\left(b^2-4c^2\right)\Bigg(\text{ArcTan}\Big[\frac{-b-2c}{\sqrt{-b^2+4ac}}\Big]\Bigg(\text{Log}\Big[1-e^{2i\left(\text{ArcTan}\left[\frac{-b-2c}{\sqrt{-b^2+4ac}}\right]+\text{ArcTan}\left[\frac{b+2cx}{\sqrt{-b^2+4ac}}\right]\right)}\Big]-$$
$$\text{Log}\Big[\text{Sin}\Big[\text{ArcTan}\Big[\frac{-b-2c}{\sqrt{-b^2+4ac}}\Big]+\text{ArcTan}\Big[\frac{b+2cx}{\sqrt{-b^2+4ac}}\Big]\Big]\Big]\Bigg)+$$
$$\text{ArcTan}\Big[\frac{-b+2c}{\sqrt{-b^2+4ac}}\Big]\Bigg(-\text{Log}\Big[1-e^{2i\left(\text{ArcTan}\left[\frac{-b+2c}{\sqrt{-b^2+4ac}}\right]+\text{ArcTan}\left[\frac{b+2cx}{\sqrt{-b^2+4ac}}\right]\right)}\Big]+$$
$$\text{Log}\Big[\text{Sin}\Big[\text{ArcTan}\Big[\frac{-b+2c}{\sqrt{-b^2+4ac}}\Big]+\text{ArcTan}\Big[\frac{b+2cx}{\sqrt{-b^2+4ac}}\Big]\Big]\Big]\Bigg)\Bigg)\Bigg)-$$
$$i\left(b^2-4c^2\right)\text{PolyLog}\Big[2,\,e^{2i\left(\text{ArcTan}\left[\frac{-b-2c}{\sqrt{-b^2+4ac}}\right]+\text{ArcTan}\left[\frac{b+2cx}{\sqrt{-b^2+4ac}}\right]\right)}\Big]+$$
$$i\left(b^2-4c^2\right)$$
$$\text{PolyLog}\Big[2,\,e^{2i\left(\text{ArcTan}\left[\frac{-b+2c}{\sqrt{-b^2+4ac}}\right]+\text{ArcTan}\left[\frac{b+2cx}{\sqrt{-b^2+4ac}}\right]\right)}\Big]\Bigg)$$

Problem 527: Unable to integrate problem.

$$\int\frac{(a+b\,\text{ArcTanh}[c\,x])\,\left(d+e\,\text{Log}\left[1-c^2x^2\right]\right)}{x}\,dx$$

Optimal (type 4, 216 leaves, 14 steps):

$$a\,d\,\mathrm{Log}[x]-\frac{1}{2}\,b\,e\,\mathrm{Log}[c\,x]\,\mathrm{Log}[1-c\,x]^2+\frac{1}{2}\,b\,e\,\mathrm{Log}[-c\,x]\,\mathrm{Log}[1+c\,x]^2-\frac{1}{2}\,b\,d\,\mathrm{PolyLog}[2,-c\,x]+\frac{1}{2}\,b\,e\,\left(\mathrm{Log}[1-c\,x]+\mathrm{Log}[1+c\,x]-\mathrm{Log}\left[1-c^2\,x^2\right]\right)\,\mathrm{PolyLog}[2,-c\,x]+\frac{1}{2}\,b\,d\,\mathrm{PolyLog}[2,c\,x]-\frac{1}{2}\,b\,e\,\left(\mathrm{Log}[1-c\,x]+\mathrm{Log}[1+c\,x]-\mathrm{Log}\left[1-c^2\,x^2\right]\right)\,\mathrm{PolyLog}[2,c\,x]-\frac{1}{2}\,a\,e\,\mathrm{PolyLog}\left[2,c^2\,x^2\right]-b\,e\,\mathrm{Log}[1-c\,x]\,\mathrm{PolyLog}[2,1-c\,x]+b\,e\,\mathrm{Log}[1+c\,x]\,\mathrm{PolyLog}[2,1+c\,x]+b\,e\,\mathrm{PolyLog}[3,1-c\,x]-b\,e\,\mathrm{PolyLog}[3,1+c\,x]$$

Result (type 8, 29 leaves):

$$\int\frac{(a+b\,\mathrm{ArcTanh}[c\,x])\,\left(d+e\,\mathrm{Log}\left[1-c^2\,x^2\right]\right)}{x}\,dx$$

Problem 528: Result more than twice size of optimal antiderivative.

$$\int\frac{(a+b\,\mathrm{ArcTanh}[c\,x])\,\left(d+e\,\mathrm{Log}\left[1-c^2\,x^2\right]\right)}{x^2}\,dx$$

Optimal (type 4, 105 leaves, 6 steps):

$$-\frac{c\,e\,(a+b\,\mathrm{ArcTanh}[c\,x])^2}{b}-\frac{(a+b\,\mathrm{ArcTanh}[c\,x])\,\left(d+e\,\mathrm{Log}\left[1-c^2\,x^2\right]\right)}{x}+\frac{1}{2}\,b\,c\,\left(d+e\,\mathrm{Log}\left[1-c^2\,x^2\right]\right)\,\mathrm{Log}\left[1-\frac{1}{1-c^2\,x^2}\right]-\frac{1}{2}\,b\,c\,e\,\mathrm{PolyLog}\left[2,\frac{1}{1-c^2\,x^2}\right]$$

Result (type 4, 332 leaves):

$$-\frac{1}{4\,x}\Bigg(4\,a\,d+4\,b\,d\,\mathrm{ArcTanh}[c\,x]+8\,a\,c\,e\,x\,\mathrm{ArcTanh}[c\,x]+4\,b\,c\,e\,x\,\mathrm{ArcTanh}[c\,x]^2-4\,b\,c\,d\,x\,\mathrm{Log}[x]-b\,c\,e\,x\,\mathrm{Log}\left[-\frac{1}{c}+x\right]^2-b\,c\,e\,x\,\mathrm{Log}\left[\frac{1}{c}+x\right]^2-2\,b\,c\,e\,x\,\mathrm{Log}\left[\frac{1}{c}+x\right]\,\mathrm{Log}\left[\frac{1}{2}\,(1-c\,x)\right]+4\,b\,c\,e\,x\,\mathrm{Log}[x]\,\mathrm{Log}[1-c\,x]-2\,b\,c\,e\,x\,\mathrm{Log}\left[-\frac{1}{c}+x\right]\,\mathrm{Log}\left[\frac{1}{2}\,(1+c\,x)\right]+4\,b\,c\,e\,x\,\mathrm{Log}[x]\,\mathrm{Log}[1+c\,x]+4\,a\,e\,\mathrm{Log}\left[1-c^2\,x^2\right]+2\,b\,c\,d\,x\,\mathrm{Log}\left[1-c^2\,x^2\right]+4\,b\,e\,\mathrm{ArcTanh}[c\,x]\,\mathrm{Log}\left[1-c^2\,x^2\right]-4\,b\,c\,e\,x\,\mathrm{Log}[x]\,\mathrm{Log}\left[1-c^2\,x^2\right]+2\,b\,c\,e\,x\,\mathrm{Log}\left[-\frac{1}{c}+x\right]\,\mathrm{Log}\left[1-c^2\,x^2\right]+2\,b\,c\,e\,x\,\mathrm{Log}\left[\frac{1}{c}+x\right]\,\mathrm{Log}\left[1-c^2\,x^2\right]+4\,b\,c\,e\,x\,\mathrm{PolyLog}[2,-c\,x]+4\,b\,c\,e\,x\,\mathrm{PolyLog}[2,c\,x]-2\,b\,c\,e\,x\,\mathrm{PolyLog}\left[2,\frac{1}{2}-\frac{c\,x}{2}\right]-2\,b\,c\,e\,x\,\mathrm{PolyLog}\left[2,\frac{1}{2}\,(1+c\,x)\right]\Bigg)$$

Problem 530: Result more than twice size of optimal antiderivative.

$$\int\frac{(a+b\,\mathrm{ArcTanh}[c\,x])\,\left(d+e\,\mathrm{Log}\left[1-c^2\,x^2\right]\right)}{x^4}\,dx$$

Optimal (type 4, 197 leaves, 15 steps):

$$\frac{2 c^2 e (a+b \text{ArcTanh}[c x])}{3 x}-\frac{c^3 e (a+b \text{ArcTanh}[c x])^2}{3 b}-b c^3 e \text{Log}[x]+\frac{1}{3} b c^3 e \text{Log}\left[1-c^2 x^2\right]-\frac{b c \left(1-c^2 x^2\right) \left(d+e \text{Log}\left[1-c^2 x^2\right]\right)}{6 x^2}-\frac{(a+b \text{ArcTanh}[c x]) \left(d+e \text{Log}\left[1-c^2 x^2\right]\right)}{3 x^3}+\frac{1}{6} b c^3 \left(d+e \text{Log}\left[1-c^2 x^2\right]\right) \text{Log}\left[1-\frac{1}{1-c^2 x^2}\right]-\frac{1}{6} b c^3 e \text{PolyLog}\left[2,\frac{1}{1-c^2 x^2}\right]$$

Result (type 4, 460 leaves):

$$\frac{1}{6} \left(-\frac{2 a d}{x^3}-\frac{b c d}{x^2}+\frac{4 a c^2 e}{x}-4 a c^3 e \text{ArcTanh}[c x]-\frac{2 b d \text{ArcTanh}[c x]}{x^3}+\frac{4 b c^2 e \text{ArcTanh}[c x]}{x}-2 b c^3 e \text{ArcTanh}[c x]^2+2 b c^3 d \text{Log}[x]-2 b c^3 e \text{Log}[x]+\frac{1}{2} b c^3 e \text{Log}\left[-\frac{1}{c}+x\right]^2+\frac{1}{2} b c^3 e \text{Log}\left[\frac{1}{c}+x\right]^2+b c^3 e \text{Log}\left[\frac{1}{c}+x\right] \text{Log}\left[\frac{1}{2} (1-c x)\right]-2 b c^3 e \text{Log}[x] \text{Log}[1-c x]+b c^3 e \text{Log}\left[-\frac{1}{c}+x\right] \text{Log}\left[\frac{1}{2} (1+c x)\right]-2 b c^3 e \text{Log}[x] \text{Log}[1+c x]-4 b c^3 e \text{Log}\left[\frac{c x}{\sqrt{1-c^2 x^2}}\right]-b c^3 d \text{Log}\left[1-c^2 x^2\right]+b c^3 e \text{Log}\left[1-c^2 x^2\right]-\frac{2 a e \text{Log}\left[1-c^2 x^2\right]}{x^3}-\frac{b c e \text{Log}\left[1-c^2 x^2\right]}{x^2}-\frac{2 b e \text{ArcTanh}[c x] \text{Log}\left[1-c^2 x^2\right]}{x^3}+2 b c^3 e \text{Log}[x] \text{Log}\left[1-c^2 x^2\right]-b c^3 e \text{Log}\left[-\frac{1}{c}+x\right] \text{Log}\left[1-c^2 x^2\right]-b c^3 e \text{Log}\left[\frac{1}{c}+x\right] \text{Log}\left[1-c^2 x^2\right]-2 b c^3 e \text{PolyLog}[2,-c x]-2 b c^3 e \text{PolyLog}[2,c x]+b c^3 e \text{PolyLog}\left[2,\frac{1}{2}-\frac{c x}{2}\right]+b c^3 e \text{PolyLog}\left[2,\frac{1}{2} (1+c x)\right]\right)$$

Problem 532: Unable to integrate problem.

$$\int \frac{(a+b \text{ArcTanh}[c x]) \left(d+e \text{Log}\left[1-c^2 x^2\right]\right)}{x^6} \, dx$$

Optimal (type 4, 256 leaves, 24 steps):

$$\frac{7 b c^3 e}{60 x^2}+\frac{2 c^2 e (a+b \text{ArcTanh}[c x])}{15 x^3}+\frac{2 c^4 e (a+b \text{ArcTanh}[c x])}{5 x}-\frac{c^5 e (a+b \text{ArcTanh}[c x])^2}{5 b}-\frac{5}{6} b c^5 e \text{Log}[x]+\frac{19}{60} b c^5 e \text{Log}\left[1-c^2 x^2\right]-\frac{b c \left(d+e \text{Log}\left[1-c^2 x^2\right]\right)}{20 x^4}-\frac{b c^3 \left(1-c^2 x^2\right) \left(d+e \text{Log}\left[1-c^2 x^2\right]\right)}{10 x^2}-\frac{(a+b \text{ArcTanh}[c x]) \left(d+e \text{Log}\left[1-c^2 x^2\right]\right)}{5 x^5}+\frac{1}{10} b c^5 \left(d+e \text{Log}\left[1-c^2 x^2\right]\right) \text{Log}\left[1-\frac{1}{1-c^2 x^2}\right]-\frac{1}{10} b c^5 e \text{PolyLog}\left[2,\frac{1}{1-c^2 x^2}\right]$$

Result (type 8, 29 leaves):

$$\int \frac{(a + b\,\text{ArcTanh}[c\,x])\,\left(d + e\,\text{Log}\left[1 - c^2\,x^2\right]\right)}{x^6}\,dx$$

Problem 533: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.

$$\int x\,(a + b\,\text{ArcTanh}[c\,x])\,\left(d + e\,\text{Log}\left[f + g\,x^2\right]\right)\,dx$$

Optimal (type 4, 512 leaves, 22 steps):

$$\frac{b\,(d-e)\,x}{2\,c} - \frac{b\,e\,x}{c} + \frac{b\,e\,\sqrt{f}\,\text{ArcTan}\left[\frac{\sqrt{g}\,x}{\sqrt{f}}\right]}{c\,\sqrt{g}} - \frac{b\,(d-e)\,\text{ArcTanh}[c\,x]}{2\,c^2} +$$
$$\frac{1}{2}\,d\,x^2\,(a + b\,\text{ArcTanh}[c\,x]) - \frac{1}{2}\,e\,x^2\,(a + b\,\text{ArcTanh}[c\,x]) -$$
$$\frac{b\,e\,\left(c^2\,f+g\right)\,\text{ArcTanh}[c\,x]\,\text{Log}\left[\frac{2}{1+c\,x}\right]}{c^2\,g} + \frac{b\,e\,\left(c^2\,f+g\right)\,\text{ArcTanh}[c\,x]\,\text{Log}\left[\frac{2\,c\,\left(\sqrt{-f}-\sqrt{g}\,x\right)}{\left(c\,\sqrt{-f}-\sqrt{g}\right)\,(1+c\,x)}\right]}{2\,c^2\,g} +$$
$$\frac{b\,e\,\left(c^2\,f+g\right)\,\text{ArcTanh}[c\,x]\,\text{Log}\left[\frac{2\,c\,\left(\sqrt{-f}+\sqrt{g}\,x\right)}{\left(c\,\sqrt{-f}+\sqrt{g}\right)\,(1+c\,x)}\right]}{2\,c^2\,g} + \frac{b\,e\,x\,\text{Log}\left[f+g\,x^2\right]}{2\,c} -$$
$$\frac{b\,e\,\left(c^2\,f+g\right)\,\text{ArcTanh}[c\,x]\,\text{Log}\left[f+g\,x^2\right]}{2\,c^2\,g} + \frac{e\,\left(f+g\,x^2\right)\,(a + b\,\text{ArcTanh}[c\,x])\,\text{Log}\left[f+g\,x^2\right]}{2\,g} +$$
$$\frac{b\,e\,\left(c^2\,f+g\right)\,\text{PolyLog}\left[2,\,1-\frac{2}{1+c\,x}\right]}{2\,c^2\,g} - \frac{b\,e\,\left(c^2\,f+g\right)\,\text{PolyLog}\left[2,\,1-\frac{2\,c\,\left(\sqrt{-f}-\sqrt{g}\,x\right)}{\left(c\,\sqrt{-f}-\sqrt{g}\right)\,(1+c\,x)}\right]}{4\,c^2\,g} -$$
$$\frac{b\,e\,\left(c^2\,f+g\right)\,\text{PolyLog}\left[2,\,1-\frac{2\,c\,\left(\sqrt{-f}+\sqrt{g}\,x\right)}{\left(c\,\sqrt{-f}+\sqrt{g}\right)\,(1+c\,x)}\right]}{4\,c^2\,g}$$

Result (type 4, 1145 leaves):

$$\frac{1}{4\,c^2\,g}\left(2\,b\,c\,d\,g\,x - 6\,b\,c\,e\,g\,x + 2\,a\,c^2\,d\,g\,x^2 - 2\,a\,c^2\,e\,g\,x^2 + 4\,b\,c\,e\,\sqrt{f}\,\sqrt{g}\,\text{ArcTan}\left[\frac{\sqrt{g}\,x}{\sqrt{f}}\right] -\right.$$
$$2\,b\,d\,g\,\text{ArcTanh}[c\,x] + 2\,b\,e\,g\,\text{ArcTanh}[c\,x] + 2\,b\,c^2\,d\,g\,x^2\,\text{ArcTanh}[c\,x] -$$
$$2\,b\,c^2\,e\,g\,x^2\,\text{ArcTanh}[c\,x] - 4\,i\,b\,c^2\,e\,f\,\text{ArcSin}\left[\sqrt{\frac{c^2\,f}{c^2\,f+g}}\right]\,\text{ArcTanh}\left[\frac{c\,g\,x}{\sqrt{-c^2\,f\,g}}\right] -$$
$$4\,i\,b\,e\,g\,\text{ArcSin}\left[\sqrt{\frac{c^2\,f}{c^2\,f+g}}\right]\,\text{ArcTanh}\left[\frac{c\,g\,x}{\sqrt{-c^2\,f\,g}}\right] - 4\,b\,c^2\,e\,f\,\text{ArcTanh}[c\,x]\,\text{Log}\left[1+e^{-2\,\text{ArcTanh}[c\,x]}\right] -$$

$$4\,b\,e\,g\,\text{ArcTanh}[c\,x]\,\text{Log}\left[1+e^{-2\,\text{ArcTanh}[c\,x]}\right]-2\,i\,b\,c^2\,e\,f\,\text{ArcSin}\left[\sqrt{\frac{c^2\,f}{c^2\,f+g}}\right]$$

$$\text{Log}\left[\frac{1}{c^2\,f+g}e^{-2\,\text{ArcTanh}[c\,x]}\left(c^2\left(1+e^{2\,\text{ArcTanh}[c\,x]}\right)f+\left(-1+e^{2\,\text{ArcTanh}[c\,x]}\right)g-2\sqrt{-c^2\,f\,g}\right)\right]-$$

$$2\,i\,b\,e\,g\,\text{ArcSin}\left[\sqrt{\frac{c^2\,f}{c^2\,f+g}}\right]$$

$$\text{Log}\left[\frac{1}{c^2\,f+g}e^{-2\,\text{ArcTanh}[c\,x]}\left(c^2\left(1+e^{2\,\text{ArcTanh}[c\,x]}\right)f+\left(-1+e^{2\,\text{ArcTanh}[c\,x]}\right)g-2\sqrt{-c^2\,f\,g}\right)\right]+$$

$$2\,b\,c^2\,e\,f\,\text{ArcTanh}[c\,x]\,\text{Log}\left[\frac{1}{c^2\,f+g}e^{-2\,\text{ArcTanh}[c\,x]}\right.$$

$$\left.\left(c^2\left(1+e^{2\,\text{ArcTanh}[c\,x]}\right)f+\left(-1+e^{2\,\text{ArcTanh}[c\,x]}\right)g-2\sqrt{-c^2\,f\,g}\right)\right]+2\,b\,e\,g\,\text{ArcTanh}[c\,x]$$

$$\text{Log}\left[\frac{1}{c^2\,f+g}e^{-2\,\text{ArcTanh}[c\,x]}\left(c^2\left(1+e^{2\,\text{ArcTanh}[c\,x]}\right)f+\left(-1+e^{2\,\text{ArcTanh}[c\,x]}\right)g-2\sqrt{-c^2\,f\,g}\right)\right]+$$

$$2\,i\,b\,c^2\,e\,f\,\text{ArcSin}\left[\sqrt{\frac{c^2\,f}{c^2\,f+g}}\right]\text{Log}\left[\frac{1}{c^2\,f+g}e^{-2\,\text{ArcTanh}[c\,x]}\right.$$

$$\left.\left(c^2\left(1+e^{2\,\text{ArcTanh}[c\,x]}\right)f+\left(-1+e^{2\,\text{ArcTanh}[c\,x]}\right)g+2\sqrt{-c^2\,f\,g}\right)\right]+2\,i\,b\,e\,g\,\text{ArcSin}\left[\sqrt{\frac{c^2\,f}{c^2\,f+g}}\right]$$

$$\text{Log}\left[\frac{1}{c^2\,f+g}e^{-2\,\text{ArcTanh}[c\,x]}\left(c^2\left(1+e^{2\,\text{ArcTanh}[c\,x]}\right)f+\left(-1+e^{2\,\text{ArcTanh}[c\,x]}\right)g+2\sqrt{-c^2\,f\,g}\right)\right]+$$

$$2\,b\,c^2\,e\,f\,\text{ArcTanh}[c\,x]\,\text{Log}\left[\frac{1}{c^2\,f+g}e^{-2\,\text{ArcTanh}[c\,x]}\right.$$

$$\left.\left(c^2\left(1+e^{2\,\text{ArcTanh}[c\,x]}\right)f+\left(-1+e^{2\,\text{ArcTanh}[c\,x]}\right)g+2\sqrt{-c^2\,f\,g}\right)\right]+2\,b\,e\,g\,\text{ArcTanh}[c\,x]$$

$$\text{Log}\left[\frac{1}{c^2\,f+g}e^{-2\,\text{ArcTanh}[c\,x]}\left(c^2\left(1+e^{2\,\text{ArcTanh}[c\,x]}\right)f+\left(-1+e^{2\,\text{ArcTanh}[c\,x]}\right)g+2\sqrt{-c^2\,f\,g}\right)\right]+$$

$$2\,a\,c^2\,e\,f\,\text{Log}\left[f+g\,x^2\right]+2\,b\,c\,e\,g\,x\,\text{Log}\left[f+g\,x^2\right]+2\,a\,c^2\,e\,g\,x^2\,\text{Log}\left[f+g\,x^2\right]-$$

$$2\,b\,e\,g\,\text{ArcTanh}[c\,x]\,\text{Log}\left[f+g\,x^2\right]+2\,b\,c^2\,e\,g\,x^2\,\text{ArcTanh}[c\,x]\,\text{Log}\left[f+g\,x^2\right]+$$

$$2\,b\,e\left(c^2\,f+g\right)\text{PolyLog}\left[2,-e^{-2\,\text{ArcTanh}[c\,x]}\right]-$$

$$b\,e\left(c^2\,f+g\right)\text{PolyLog}\left[2,\frac{e^{-2\,\text{ArcTanh}[c\,x]}\left(-c^2\,f+g-2\sqrt{-c^2\,f\,g}\right)}{c^2\,f+g}\right]-$$

$$b\,c^2\,e\,f\,\text{PolyLog}\left[2,\frac{e^{-2\,\text{ArcTanh}[c\,x]}\left(-c^2\,f+g+2\sqrt{-c^2\,f\,g}\right)}{c^2\,f+g}\right]-$$

$$\left.b\,e\,g\,\text{PolyLog}\left[2,\frac{e^{-2\,\text{ArcTanh}[c\,x]}\left(-c^2\,f+g+2\sqrt{-c^2\,f\,g}\right)}{c^2\,f+g}\right]\right)$$

Problem 534: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.

$$\int (a + b\,\text{ArcTanh}[c\,x])\,\left(d + e\,\text{Log}\left[f + g\,x^2\right]\right)\,dx$$

Optimal (type 4, 599 leaves, 28 steps):

$$-2\,a\,e\,x + \frac{2\,a\,e\,\sqrt{f}\,\text{ArcTan}\left[\frac{\sqrt{g}\,x}{\sqrt{f}}\right]}{\sqrt{g}} - 2\,b\,e\,x\,\text{ArcTanh}[c\,x] + \frac{b\,e\,\sqrt{-f}\,\text{Log}[1-c\,x]\,\text{Log}\left[\frac{c\left(\sqrt{-f}-\sqrt{g}\,x\right)}{c\,\sqrt{-f}-\sqrt{g}}\right]}{2\,\sqrt{g}} -$$
$$\frac{b\,e\,\sqrt{-f}\,\text{Log}[1+c\,x]\,\text{Log}\left[\frac{c\left(\sqrt{-f}-\sqrt{g}\,x\right)}{c\,\sqrt{-f}+\sqrt{g}}\right]}{2\,\sqrt{g}} + \frac{b\,e\,\sqrt{-f}\,\text{Log}[1+c\,x]\,\text{Log}\left[\frac{c\left(\sqrt{-f}+\sqrt{g}\,x\right)}{c\,\sqrt{-f}-\sqrt{g}}\right]}{2\,\sqrt{g}} -$$
$$\frac{b\,e\,\sqrt{-f}\,\text{Log}[1-c\,x]\,\text{Log}\left[\frac{c\left(\sqrt{-f}+\sqrt{g}\,x\right)}{c\,\sqrt{-f}+\sqrt{g}}\right]}{2\,\sqrt{g}} - \frac{b\,e\,\text{Log}\left[1-c^2\,x^2\right]}{c} +$$
$$x\,(a + b\,\text{ArcTanh}[c\,x])\,\left(d + e\,\text{Log}\left[f + g\,x^2\right]\right) + \frac{b\,\text{Log}\left[\frac{g\,\left(1-c^2\,x^2\right)}{c^2\,f+g}\right]\,\left(d + e\,\text{Log}\left[f + g\,x^2\right]\right)}{2\,c} +$$
$$\frac{b\,e\,\sqrt{-f}\,\text{PolyLog}\left[2,\,-\frac{\sqrt{g}\,(1-c\,x)}{c\,\sqrt{-f}-\sqrt{g}}\right]}{2\,\sqrt{g}} - \frac{b\,e\,\sqrt{-f}\,\text{PolyLog}\left[2,\,\frac{\sqrt{g}\,(1-c\,x)}{c\,\sqrt{-f}+\sqrt{g}}\right]}{2\,\sqrt{g}} +$$
$$\frac{b\,e\,\sqrt{-f}\,\text{PolyLog}\left[2,\,-\frac{\sqrt{g}\,(1+c\,x)}{c\,\sqrt{-f}-\sqrt{g}}\right]}{2\,\sqrt{g}} - \frac{b\,e\,\sqrt{-f}\,\text{PolyLog}\left[2,\,\frac{\sqrt{g}\,(1+c\,x)}{c\,\sqrt{-f}+\sqrt{g}}\right]}{2\,\sqrt{g}} + \frac{b\,e\,\text{PolyLog}\left[2,\,\frac{c^2\,\left(f+g\,x^2\right)}{c^2\,f+g}\right]}{2\,c}$$

Result (type 4, 1251 leaves):

$$a\,d\,x - 2\,a\,e\,x + \frac{2\,a\,e\,\sqrt{f}\,\text{ArcTan}\left[\frac{\sqrt{g}\,x}{\sqrt{f}}\right]}{\sqrt{g}} + b\,d\,x\,\text{ArcTanh}[c\,x] + \frac{b\,d\,\text{Log}\left[1-c^2\,x^2\right]}{2\,c} +$$
$$a\,e\,x\,\text{Log}\left[f+g\,x^2\right] + b\,e\left(x\,\text{ArcTanh}[c\,x] + \frac{\text{Log}\left[1-c^2\,x^2\right]}{2\,c}\right)\,\text{Log}\left[f+g\,x^2\right] -$$
$$\frac{1}{c}b\,e\,g\left(\frac{\left(-\text{Log}\left[-\frac{1}{c}+x\right] - \text{Log}\left[\frac{1}{c}+x\right] + \text{Log}\left[1-c^2\,x^2\right]\right)\,\text{Log}\left[f+g\,x^2\right]}{2\,g} +\right.$$
$$\frac{\text{Log}\left[-\frac{1}{c}+x\right]\,\text{Log}\left[1-\frac{\sqrt{g}\,\left(-\frac{1}{c}+x\right)}{-i\,\sqrt{f}-\frac{\sqrt{g}}{c}}\right] + \text{PolyLog}\left[2,\,\frac{\sqrt{g}\,\left(-\frac{1}{c}+x\right)}{-i\,\sqrt{f}-\frac{\sqrt{g}}{c}}\right]}{2\,g} +$$

$$\frac{\text{Log}\left[-\frac{1}{c}+x\right]\,\text{Log}\left[1-\frac{\sqrt{g}\,\left(-\frac{1}{c}+x\right)}{\mathrm{i}\,\sqrt{f}-\frac{\sqrt{g}}{c}}\right]+\text{PolyLog}\left[2,\,\frac{\sqrt{g}\,\left(-\frac{1}{c}+x\right)}{\mathrm{i}\,\sqrt{f}-\frac{\sqrt{g}}{c}}\right]}{2\,g}+$$

$$\frac{\text{Log}\left[\frac{1}{c}+x\right]\,\text{Log}\left[1-\frac{\sqrt{g}\,\left(\frac{1}{c}+x\right)}{-\mathrm{i}\,\sqrt{f}+\frac{\sqrt{g}}{c}}\right]+\text{PolyLog}\left[2,\,\frac{\sqrt{g}\,\left(\frac{1}{c}+x\right)}{-\mathrm{i}\,\sqrt{f}+\frac{\sqrt{g}}{c}}\right]}{2\,g}+$$

$$\left.\frac{\text{Log}\left[\frac{1}{c}+x\right]\,\text{Log}\left[1-\frac{\sqrt{g}\,\left(\frac{1}{c}+x\right)}{\mathrm{i}\,\sqrt{f}+\frac{\sqrt{g}}{c}}\right]+\text{PolyLog}\left[2,\,\frac{\sqrt{g}\,\left(\frac{1}{c}+x\right)}{\mathrm{i}\,\sqrt{f}+\frac{\sqrt{g}}{c}}\right]}{2\,g}\right)-$$

$$\frac{1}{2\,c}\,b\,e\,\left(4\,c\,x\,\text{ArcTanh}[c\,x]-4\,\text{Log}\left[\frac{1}{\sqrt{1-c^2\,x^2}}\right]+\right.$$

$$\frac{1}{g}\,\sqrt{c^2\,f\,g}\,\left(-2\,\mathrm{i}\,\text{ArcCos}\left[\frac{-c^2\,f+g}{c^2\,f+g}\right]\,\text{ArcTan}\left[\frac{c\,g\,x}{\sqrt{c^2\,f\,g}}\right]+4\,\text{ArcTan}\left[\frac{\sqrt{c^2\,f\,g}}{c\,g\,x}\right]\,\text{ArcTanh}[c\,x]-\right.$$

$$\left(\text{ArcCos}\left[\frac{-c^2\,f+g}{c^2\,f+g}\right]-2\,\text{ArcTan}\left[\frac{c\,g\,x}{\sqrt{c^2\,f\,g}}\right]\right)\,\text{Log}\left[\frac{2\,c^2\,f\,\left(g+\mathrm{i}\,\sqrt{c^2\,f\,g}\right)\,\left(1+c\,x\right)}{\left(c^2\,f+g\right)\,\left(c^2\,f+\mathrm{i}\,c\,\sqrt{c^2\,f\,g}\,x\right)}\right]-$$

$$\left(\text{ArcCos}\left[\frac{-c^2\,f+g}{c^2\,f+g}\right]+2\,\text{ArcTan}\left[\frac{c\,g\,x}{\sqrt{c^2\,f\,g}}\right]\right)\,\text{Log}\left[\frac{2\,c^2\,f\,\left(\mathrm{i}\,g+\sqrt{c^2\,f\,g}\right)\,\left(-1+c\,x\right)}{\left(c^2\,f+g\right)\,\left(-\mathrm{i}\,c^2\,f+c\,\sqrt{c^2\,f\,g}\,x\right)}\right]+$$

$$\left(\text{ArcCos}\left[\frac{-c^2\,f+g}{c^2\,f+g}\right]+2\,\left(\text{ArcTan}\left[\frac{\sqrt{c^2\,f\,g}}{c\,g\,x}\right]+\text{ArcTan}\left[\frac{c\,g\,x}{\sqrt{c^2\,f\,g}}\right]\right)\right)$$

$$\text{Log}\left[\frac{\sqrt{2}\,e^{-\text{ArcTanh}[c\,x]}\,\sqrt{c^2\,f\,g}}{\sqrt{c^2\,f+g}\,\sqrt{c^2\,f-g+\left(c^2\,f+g\right)\,\text{Cosh}[2\,\text{ArcTanh}[c\,x]]}}\right]+$$

$$\left(\text{ArcCos}\left[\frac{-c^2\,f+g}{c^2\,f+g}\right]-2\,\left(\text{ArcTan}\left[\frac{\sqrt{c^2\,f\,g}}{c\,g\,x}\right]+\text{ArcTan}\left[\frac{c\,g\,x}{\sqrt{c^2\,f\,g}}\right]\right)\right)$$

$$\text{Log}\left[\frac{\sqrt{2}\,e^{\text{ArcTanh}[c\,x]}\,\sqrt{c^2\,f\,g}}{\sqrt{c^2\,f+g}\,\sqrt{c^2\,f-g+\left(c^2\,f+g\right)\,\text{Cosh}[2\,\text{ArcTanh}[c\,x]]}}\right]+$$

$$\mathrm{i}\left(-\text{PolyLog}\left[2,\frac{\left(-c^2 f+g-2\,\mathrm{i}\sqrt{c^2 f g}\right)\left(\mathrm{i}\,c^2 f+c\sqrt{c^2 f g}\;x\right)}{\left(c^2 f+g\right)\left(-\mathrm{i}\,c^2 f+c\sqrt{c^2 f g}\;x\right)}\right]+\text{PolyLog}\left[2,\frac{\left(-c^2 f+g+2\,\mathrm{i}\sqrt{c^2 f g}\right)\left(\mathrm{i}\,c^2 f+c\sqrt{c^2 f g}\;x\right)}{\left(c^2 f+g\right)\left(-\mathrm{i}\,c^2 f+c\sqrt{c^2 f g}\;x\right)}\right]\right)\Bigg)\Bigg)$$

Problem 536: Result unnecessarily involves imaginary or complex numbers.

$$\int\frac{\left(a+b\,\text{ArcTanh}[c\,x]\right)\left(d+e\,\text{Log}\left[f+g\,x^2\right]\right)}{x^2}\,dx$$

Optimal (type 4, 613 leaves, 28 steps):

$$\frac{2\,a\,e\sqrt{g}\,\text{ArcTan}\left[\frac{\sqrt{g}\,x}{\sqrt{f}}\right]}{\sqrt{f}}-\frac{b\,e\sqrt{g}\,\text{Log}[1-c\,x]\,\text{Log}\left[\frac{c\left(\sqrt{-f}-\sqrt{g}\,x\right)}{c\sqrt{-f}-\sqrt{g}}\right]}{2\sqrt{-f}}+$$
$$\frac{b\,e\sqrt{g}\,\text{Log}[1+c\,x]\,\text{Log}\left[\frac{c\left(\sqrt{-f}-\sqrt{g}\,x\right)}{c\sqrt{-f}+\sqrt{g}}\right]}{2\sqrt{-f}}-\frac{b\,e\sqrt{g}\,\text{Log}[1+c\,x]\,\text{Log}\left[\frac{c\left(\sqrt{-f}+\sqrt{g}\,x\right)}{c\sqrt{-f}-\sqrt{g}}\right]}{2\sqrt{-f}}+$$
$$\frac{b\,e\sqrt{g}\,\text{Log}[1-c\,x]\,\text{Log}\left[\frac{c\left(\sqrt{-f}+\sqrt{g}\,x\right)}{c\sqrt{-f}+\sqrt{g}}\right]}{2\sqrt{-f}}-\frac{\left(a+b\,\text{ArcTanh}[c\,x]\right)\left(d+e\,\text{Log}\left[f+g\,x^2\right]\right)}{x}+$$
$$\frac{1}{2}\,b\,c\,\text{Log}\left[-\frac{g\,x^2}{f}\right]\left(d+e\,\text{Log}\left[f+g\,x^2\right]\right)-\frac{1}{2}\,b\,c\,\text{Log}\left[\frac{g\left(1-c^2 x^2\right)}{c^2 f+g}\right]\left(d+e\,\text{Log}\left[f+g\,x^2\right]\right)-$$
$$\frac{b\,e\sqrt{g}\,\text{PolyLog}\left[2,-\frac{\sqrt{g}\,(1-c\,x)}{c\sqrt{-f}-\sqrt{g}}\right]}{2\sqrt{-f}}+\frac{b\,e\sqrt{g}\,\text{PolyLog}\left[2,\frac{\sqrt{g}\,(1-c\,x)}{c\sqrt{-f}+\sqrt{g}}\right]}{2\sqrt{-f}}-$$
$$\frac{b\,e\sqrt{g}\,\text{PolyLog}\left[2,-\frac{\sqrt{g}\,(1+c\,x)}{c\sqrt{-f}-\sqrt{g}}\right]}{2\sqrt{-f}}+\frac{b\,e\sqrt{g}\,\text{PolyLog}\left[2,\frac{\sqrt{g}\,(1+c\,x)}{c\sqrt{-f}+\sqrt{g}}\right]}{2\sqrt{-f}}-$$
$$\frac{1}{2}\,b\,c\,e\,\text{PolyLog}\left[2,\frac{c^2\left(f+g\,x^2\right)}{c^2 f+g}\right]+\frac{1}{2}\,b\,c\,e\,\text{PolyLog}\left[2,1+\frac{g\,x^2}{f}\right]$$

Result (type 4, 1226 leaves):

$$-\frac{a\,d}{x}-\frac{b\,d\,\text{ArcTanh}[c\,x]}{x}+b\,c\,d\,\text{Log}[x]-$$
$$\frac{1}{2}\,b\,c\,d\,\text{Log}\left[1-c^2 x^2\right]+a\,e\left(\frac{2\sqrt{g}\,\text{ArcTan}\left[\frac{\sqrt{g}\,x}{\sqrt{f}}\right]}{\sqrt{f}}-\frac{\text{Log}\left[f+g\,x^2\right]}{x}\right)+$$

$$\frac{1}{2} b e \left(-\frac{\left(2 \text{ArcTanh}[c x]+c x \left(-2 \text{Log}[x]+\text{Log}\left[1-c^2 x^2\right]\right)\right) \text{Log}\left[f+g x^2\right]}{x}-2 c \left(\text{Log}[x]\right.\right.$$

$$\left.\left(\text{Log}\left[1-\frac{\mathbb{i} \sqrt{g} x}{\sqrt{f}}\right]+\text{Log}\left[1+\frac{\mathbb{i} \sqrt{g} x}{\sqrt{f}}\right]\right)+\text{PolyLog}\left[2,-\frac{\mathbb{i} \sqrt{g} x}{\sqrt{f}}\right]+\text{PolyLog}\left[2,\frac{\mathbb{i} \sqrt{g} x}{\sqrt{f}}\right]\right)+$$

$$c \left(\text{Log}\left[-\frac{1}{c}+x\right] \text{Log}\left[\frac{c \left(\sqrt{f}-\mathbb{i} \sqrt{g} x\right)}{c \sqrt{f}-\mathbb{i} \sqrt{g}}\right]+\text{Log}\left[\frac{1}{c}+x\right] \text{Log}\left[\frac{c \left(\sqrt{f}-\mathbb{i} \sqrt{g} x\right)}{c \sqrt{f}+\mathbb{i} \sqrt{g}}\right]+\right.$$

$$\text{Log}\left[-\frac{1}{c}+x\right] \text{Log}\left[\frac{c \left(\sqrt{f}+\mathbb{i} \sqrt{g} x\right)}{c \sqrt{f}+\mathbb{i} \sqrt{g}}\right]-\left(\text{Log}\left[-\frac{1}{c}+x\right]+\text{Log}\left[\frac{1}{c}+x\right]-\text{Log}\left[1-c^2 x^2\right]\right)$$

$$\text{Log}\left[f+g x^2\right]+\text{Log}\left[\frac{1}{c}+x\right] \text{Log}\left[1-\frac{\sqrt{g} (1+c x)}{\mathbb{i} c \sqrt{f}+\sqrt{g}}\right]+\text{PolyLog}\left[2,\frac{c \sqrt{g} \left(\frac{1}{c}+x\right)}{\mathbb{i} c \sqrt{f}+\sqrt{g}}\right]+\text{PolyLog}\left[\right.$$

$$\left.\left.2,\frac{\mathbb{i} \sqrt{g} (-1+c x)}{c \sqrt{f}-\mathbb{i} \sqrt{g}}\right]+\text{PolyLog}\left[2,-\frac{\mathbb{i} \sqrt{g} (-1+c x)}{c \sqrt{f}+\mathbb{i} \sqrt{g}}\right]+\text{PolyLog}\left[2,\frac{\mathbb{i} \sqrt{g} (1+c x)}{c \sqrt{f}+\mathbb{i} \sqrt{g}}\right]\right)+$$

$$\frac{1}{\sqrt{c^2 f g}} c g \left(2 \mathbb{i} \text{ArcCos}\left[\frac{-c^2 f+g}{c^2 f+g}\right] \text{ArcTan}\left[\frac{c g x}{\sqrt{c^2 f g}}\right]-4 \text{ArcTan}\left[\frac{c f}{\sqrt{c^2 f g} x}\right] \text{ArcTanh}[c x]+\right.$$

$$\left(\text{ArcCos}\left[\frac{-c^2 f+g}{c^2 f+g}\right]+2 \text{ArcTan}\left[\frac{c g x}{\sqrt{c^2 f g}}\right]\right) \text{Log}\left[\frac{2 \mathbb{i} c f \left(\mathbb{i} g+\sqrt{c^2 f g}\right) (-1+c x)}{\left(c^2 f+g\right) \left(c f+\mathbb{i} \sqrt{c^2 f g} x\right)}\right]+$$

$$\left(\text{ArcCos}\left[\frac{-c^2 f+g}{c^2 f+g}\right]-2 \text{ArcTan}\left[\frac{c g x}{\sqrt{c^2 f g}}\right]\right) \text{Log}\left[\frac{2 c f \left(g+\mathbb{i} \sqrt{c^2 f g}\right) (1+c x)}{\left(c^2 f+g\right) \left(c f+\mathbb{i} \sqrt{c^2 f g} x\right)}\right]-$$

$$\left(\text{ArcCos}\left[\frac{-c^2 f+g}{c^2 f+g}\right]+2 \left(\text{ArcTan}\left[\frac{c f}{\sqrt{c^2 f g} x}\right]+\text{ArcTan}\left[\frac{c g x}{\sqrt{c^2 f g}}\right]\right)\right)$$

$$\text{Log}\left[\frac{\sqrt{2} e^{-\text{ArcTanh}[c x]} \sqrt{c^2 f g}}{\sqrt{c^2 f+g} \sqrt{c^2 f-g+\left(c^2 f+g\right) \text{Cosh}[2 \text{ArcTanh}[c x]]}}\right]-$$

$$\left(\text{ArcCos}\left[\frac{-c^2 f+g}{c^2 f+g}\right]-2 \left(\text{ArcTan}\left[\frac{c f}{\sqrt{c^2 f g} x}\right]+\text{ArcTan}\left[\frac{c g x}{\sqrt{c^2 f g}}\right]\right)\right)$$

$$\text{Log}\left[\frac{\sqrt{2} e^{\text{ArcTanh}[c x]} \sqrt{c^2 f g}}{\sqrt{c^2 f+g} \sqrt{c^2 f-g+\left(c^2 f+g\right) \text{Cosh}[2 \text{ArcTanh}[c x]]}}\right]+$$

$$\mathrm{i}\left(\text{PolyLog}\left[2,\frac{\left(-c^2 f+g-2\,\mathrm{i}\,\sqrt{c^2 f g}\right)\left(\mathrm{i}\,c\,f+\sqrt{c^2 f g}\;x\right)}{\left(c^2 f+g\right)\left(-\mathrm{i}\,c\,f+\sqrt{c^2 f g}\;x\right)}\right]-\right.$$

$$\left.\left.\left.\text{PolyLog}\left[2,\frac{\left(-c^2 f+g+2\,\mathrm{i}\,\sqrt{c^2 f g}\right)\left(\mathrm{i}\,c\,f+\sqrt{c^2 f g}\;x\right)}{\left(c^2 f+g\right)\left(-\mathrm{i}\,c\,f+\sqrt{c^2 f g}\;x\right)}\right]\right)\right)\right)$$

Problem 537: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.

$$\int \frac{\left(a+b\,\text{ArcTanh}[c\,x]\right)\left(d+e\,\text{Log}\left[f+g\,x^2\right]\right)}{x^3}\,dx$$

Optimal (type 4, 470 leaves, 20 steps):

$$\frac{b\,c\,e\,\sqrt{g}\;\text{ArcTan}\left[\frac{\sqrt{g}\;x}{\sqrt{f}}\right]}{\sqrt{f}}+\frac{a\,e\,g\,\text{Log}[x]}{f}+\frac{b\,e\,\left(c^2 f+g\right)\text{ArcTanh}[c\,x]\;\text{Log}\left[\frac{2}{1+c\,x}\right]}{f}-$$

$$\frac{b\,e\,\left(c^2 f+g\right)\text{ArcTanh}[c\,x]\;\text{Log}\left[\frac{2\,c\left(\sqrt{-f}-\sqrt{g}\;x\right)}{\left(c\,\sqrt{-f}-\sqrt{g}\right)(1+c\,x)}\right]}{2\,f}-$$

$$\frac{b\,e\,\left(c^2 f+g\right)\text{ArcTanh}[c\,x]\;\text{Log}\left[\frac{2\,c\left(\sqrt{-f}+\sqrt{g}\;x\right)}{\left(c\,\sqrt{-f}+\sqrt{g}\right)(1+c\,x)}\right]}{2\,f}-\frac{a\,e\,g\,\text{Log}\left[f+g\,x^2\right]}{2\,f}-\frac{b\,c\left(d+e\,\text{Log}\left[f+g\,x^2\right]\right)}{2\,x}+$$

$$\frac{1}{2}\,b\,c^2\,\text{ArcTanh}[c\,x]\left(d+e\,\text{Log}\left[f+g\,x^2\right]\right)-\frac{\left(a+b\,\text{ArcTanh}[c\,x]\right)\left(d+e\,\text{Log}\left[f+g\,x^2\right]\right)}{2\,x^2}-$$

$$\frac{b\,e\,g\,\text{PolyLog}[2,-c\,x]}{2\,f}+\frac{b\,e\,g\,\text{PolyLog}[2,c\,x]}{2\,f}-\frac{b\,e\,\left(c^2 f+g\right)\text{PolyLog}\left[2,1-\frac{2}{1+c\,x}\right]}{2\,f}+$$

$$\frac{b\,e\,\left(c^2 f+g\right)\text{PolyLog}\left[2,1-\frac{2\,c\left(\sqrt{-f}-\sqrt{g}\;x\right)}{\left(c\,\sqrt{-f}-\sqrt{g}\right)(1+c\,x)}\right]}{4\,f}+\frac{b\,e\,\left(c^2 f+g\right)\text{PolyLog}\left[2,1-\frac{2\,c\left(\sqrt{-f}+\sqrt{g}\;x\right)}{\left(c\,\sqrt{-f}+\sqrt{g}\right)(1+c\,x)}\right]}{4\,f}$$

Result (type 4, 1211 leaves):

$$\frac{1}{4\,f\,x^2}\left(-2\,a\,d\,f-2\,b\,c\,d\,f\,x+4\,b\,c\,e\,\sqrt{f}\;\sqrt{g}\;x^2\,\text{ArcTan}\left[\frac{\sqrt{g}\;x}{\sqrt{f}}\right]-2\,b\,d\,f\,\text{ArcTanh}[c\,x]+\right.$$

$$2\,b\,c^2\,d\,f\,x^2\,\text{ArcTanh}[c\,x]+4\,\mathrm{i}\,b\,c^2\,e\,f\,x^2\,\text{ArcSin}\left[\sqrt{\frac{c^2 f}{c^2 f+g}}\right]\text{ArcTanh}\left[\frac{c\,g\,x}{\sqrt{-c^2 f\,g}}\right]+$$

$$4\,i\,b\,e\,g\,x^2\,\text{ArcSin}\left[\sqrt{\frac{c^2 f}{c^2 f+g}}\right]\,\text{ArcTanh}\left[\frac{c\,g\,x}{\sqrt{-c^2 f g}}\right]+$$

$$4\,b\,e\,g\,x^2\,\text{ArcTanh}[c\,x]\,\text{Log}\left[1-e^{-2\,\text{ArcTanh}[c\,x]}\right]+$$

$$4\,b\,c^2\,e\,f\,x^2\,\text{ArcTanh}[c\,x]\,\text{Log}\left[1+e^{-2\,\text{ArcTanh}[c\,x]}\right]+2\,i\,b\,c^2\,e\,f\,x^2\,\text{ArcSin}\left[\sqrt{\frac{c^2 f}{c^2 f+g}}\right]$$

$$\text{Log}\left[\frac{1}{c^2 f+g}e^{-2\,\text{ArcTanh}[c\,x]}\left(c^2\left(1+e^{2\,\text{ArcTanh}[c\,x]}\right)f+\left(-1+e^{2\,\text{ArcTanh}[c\,x]}\right)g-2\sqrt{-c^2 f g}\right)\right]+$$

$$2\,i\,b\,e\,g\,x^2\,\text{ArcSin}\left[\sqrt{\frac{c^2 f}{c^2 f+g}}\right]$$

$$\text{Log}\left[\frac{1}{c^2 f+g}e^{-2\,\text{ArcTanh}[c\,x]}\left(c^2\left(1+e^{2\,\text{ArcTanh}[c\,x]}\right)f+\left(-1+e^{2\,\text{ArcTanh}[c\,x]}\right)g-2\sqrt{-c^2 f g}\right)\right]-$$

$$2\,b\,c^2\,e\,f\,x^2\,\text{ArcTanh}[c\,x]\,\text{Log}\left[\frac{1}{c^2 f+g}e^{-2\,\text{ArcTanh}[c\,x]}\right.$$

$$\left.\left(c^2\left(1+e^{2\,\text{ArcTanh}[c\,x]}\right)f+\left(-1+e^{2\,\text{ArcTanh}[c\,x]}\right)g-2\sqrt{-c^2 f g}\right)\right]-2\,b\,e\,g\,x^2\,\text{ArcTanh}[c\,x]$$

$$\text{Log}\left[\frac{1}{c^2 f+g}e^{-2\,\text{ArcTanh}[c\,x]}\left(c^2\left(1+e^{2\,\text{ArcTanh}[c\,x]}\right)f+\left(-1+e^{2\,\text{ArcTanh}[c\,x]}\right)g-2\sqrt{-c^2 f g}\right)\right]-$$

$$2\,i\,b\,c^2\,e\,f\,x^2\,\text{ArcSin}\left[\sqrt{\frac{c^2 f}{c^2 f+g}}\right]$$

$$\text{Log}\left[\frac{1}{c^2 f+g}e^{-2\,\text{ArcTanh}[c\,x]}\left(c^2\left(1+e^{2\,\text{ArcTanh}[c\,x]}\right)f+\left(-1+e^{2\,\text{ArcTanh}[c\,x]}\right)g+2\sqrt{-c^2 f g}\right)\right]-$$

$$2\,i\,b\,e\,g\,x^2\,\text{ArcSin}\left[\sqrt{\frac{c^2 f}{c^2 f+g}}\right]$$

$$\text{Log}\left[\frac{1}{c^2 f+g}e^{-2\,\text{ArcTanh}[c\,x]}\left(c^2\left(1+e^{2\,\text{ArcTanh}[c\,x]}\right)f+\left(-1+e^{2\,\text{ArcTanh}[c\,x]}\right)g+2\sqrt{-c^2 f g}\right)\right]-$$

$$2\,b\,c^2\,e\,f\,x^2\,\text{ArcTanh}[c\,x]\,\text{Log}\left[\frac{1}{c^2 f+g}e^{-2\,\text{ArcTanh}[c\,x]}\right.$$

$$\left.\left(c^2\left(1+e^{2\,\text{ArcTanh}[c\,x]}\right)f+\left(-1+e^{2\,\text{ArcTanh}[c\,x]}\right)g+2\sqrt{-c^2 f g}\right)\right]-2\,b\,e\,g\,x^2\,\text{ArcTanh}[c\,x]$$

$$\text{Log}\left[\frac{1}{c^2 f+g}e^{-2\,\text{ArcTanh}[c\,x]}\left(c^2\left(1+e^{2\,\text{ArcTanh}[c\,x]}\right)f+\left(-1+e^{2\,\text{ArcTanh}[c\,x]}\right)g+2\sqrt{-c^2 f g}\right)\right]+$$

$$4\,a\,e\,g\,x^2\,\text{Log}[x]-2\,a\,e\,f\,\text{Log}\left[f+g\,x^2\right]-2\,b\,c\,e\,f\,x\,\text{Log}\left[f+g\,x^2\right]-2\,a\,e\,g\,x^2\,\text{Log}\left[f+g\,x^2\right]-$$

$$2\,b\,e\,f\,\text{ArcTanh}[c\,x]\,\text{Log}\left[f+g\,x^2\right]+2\,b\,c^2\,e\,f\,x^2\,\text{ArcTanh}[c\,x]\,\text{Log}\left[f+g\,x^2\right]-$$

$$2\,b\,c^2\,e\,f\,x^2\,\text{PolyLog}\left[2,-e^{-2\,\text{ArcTanh}[c\,x]}\right]-2\,b\,e\,g\,x^2\,\text{PolyLog}\left[2,e^{-2\,\text{ArcTanh}[c\,x]}\right]+$$

$$b\,c^2\,e\,f\,x^2\,\text{PolyLog}\left[2,\frac{e^{-2\,\text{ArcTanh}[c\,x]}\left(-c^2 f+g-2\sqrt{-c^2 f g}\right)}{c^2 f+g}\right]+$$

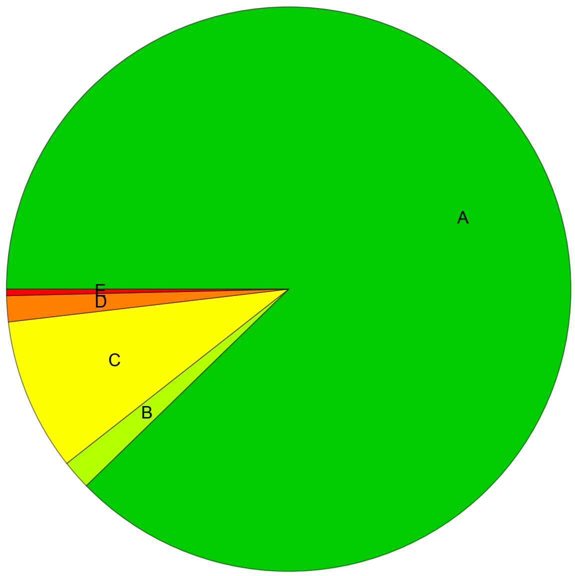
$$b\,e\,g\,x^2\,\text{PolyLog}\left[2,\ \frac{e^{-2\,\text{ArcTanh}[c\,x]}\left(-c^2\,f+g-2\,\sqrt{-c^2\,f\,g}\right)}{c^2\,f+g}\right]+$$

$$b\,c^2\,e\,f\,x^2\,\text{PolyLog}\left[2,\ \frac{e^{-2\,\text{ArcTanh}[c\,x]}\left(-c^2\,f+g+2\,\sqrt{-c^2\,f\,g}\right)}{c^2\,f+g}\right]+$$

$$b\,e\,g\,x^2\,\text{PolyLog}\left[2,\ \frac{e^{-2\,\text{ArcTanh}[c\,x]}\left(-c^2\,f+g+2\,\sqrt{-c^2\,f\,g}\right)}{c^2\,f+g}\right]\Bigg)$$

Summary of Integration Test Results

538 integration problems



A - 472 optimal antiderivatives

B - 9 more than twice size of optimal antiderivatives

C - 47 unnecessarily complex antiderivatives

D - 8 unable to integrate problems

E - 2 integration timeouts